Theorems on Transform for Product of Generalized M-Series, I-Function of Two Variables and It's Applications

Vinita Jain, Ashok Kumar Ronghe, Shailesh Jaloree, and Anil Goyal

Abstract — In the present paper we establish four theorem which involves I-function of two variables and generalized M-series. In next section we obtain certain new integrals by application of our theorems by giving suitable values to the parameters. Then main theorem reduces to H-function of two variables etc.

Keywords — Integral transforms, I-Function, Hypergeometric series, M-series.

I. INTRODUCTION

The I-function of two variables represented by means of two contour integrals is defined by Goyal and Agarwal [1] are represented in the following manner :

$$\mathbf{I}_{p,q;p_1^{(1)},q_1^{(1)},p_1^{(2)},q_1^{(2)};r} \left[\begin{matrix} z_1 \left[\left(\epsilon_p : E_p, E'_p \right) : \left(a_j, \alpha_j \right)_{1,n_2} \right], \left(a_{ji}, \alpha_{ji} \right)_{n_2+1,p_1^{(1)}} \right] ; \left(c_{ji}, \gamma_{ji} \right)_{1,n_3} \\ z_2 \left[\left(f_q : f_q, f'_q \right) : \left(b_j, \beta_j \right)_{1,m_2} \right], \left(b_{ji}, \beta_{ji} \right)_{m_2+1,q_1^{(1)}} \right] ; \left(d_{ji}, \delta_{ji} \right)_{1,m_3} \\ \left(c_{ji}, \gamma_{ji} \right)_{n_3+1,p_1^{(2)}} \\ \left(d_{ji}, \delta_{ji} \right)_{m_3+1,q_1^{(2)}} \end{matrix} \right]$$

or

$$\mathbf{I}_{p,q;Q;r} \left[\begin{matrix} z_1 \left[((P)), ((Q)), ((R)), ((S)), ((T)) \right] \\ z_2 \left[((U)), ((V)), ((W)), ((X)), ((Y)) \right] \end{matrix} \right] = \frac{1}{(2\pi\omega)} \int_{L_1} \int_{L_2} \phi_1(\xi) \cdot \phi_2(\eta) \cdot \psi(\xi\eta) z_1^\xi \cdot z_2^\eta d\xi \cdot d\eta \quad (1)$$

$$\phi_1(\xi) = \frac{\prod_{j=1}^{m_2} \Gamma(b_j - \beta_j \xi) \cdot \prod_{j=1}^{n_2} \Gamma(1 - a_j + \alpha_j \xi)}{\sum_{i=1}^r \left[\prod_{j=m_2+1}^{q_i^{(1)}} \Gamma(1 - b_{ji} + \beta_{ji} \xi) \cdot \prod_{j=n_2+1}^{p_i^{(1)}} \Gamma(a_{ji} - \alpha_{ji} \xi) \right]} \quad (2)$$

$$\phi_2(\eta) = \frac{\prod_{j=1}^{m_3} \Gamma(d_j - \delta_j \eta) \cdot \prod_{j=1}^{n_3} \Gamma(1 - c_j + \gamma_j \eta)}{\sum_{i=1}^r \left[\prod_{j=m_2+1}^{q_i^{(2)}} \Gamma(1 - d_{ji} + \delta_{ji} \eta) \cdot \prod_{j=n_2+1}^{p_i^{(2)}} \Gamma(c_{ji} - \gamma_{ji} \eta) \right]} \quad (3)$$

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$$\psi(\xi, \eta) = \frac{\prod_{j=1}^{m_1} \Gamma(f_j - F_j \xi - F_j \eta) \cdot \prod_{j=1}^{n_1} \Gamma(1 - e_j + E_j \xi + E_j \eta)}{\prod_{j=m_1+1}^q \Gamma(1 - f_j \xi + F_j \eta) \cdot \prod_{j=n_1+1}^p \Gamma(e_j - E_j \xi - E_j \eta)} \quad (4)$$

z_1 and z_2 are two non zero complex variables. L_1 and L_2 are double Mellin-Barnes type contour integrals and convergence condition are as follow :

$$|\arg(z_1)| < \frac{\pi A_i}{2}, \quad |\arg(z_2)| < \frac{\pi B_i}{2} \quad (5)$$

where,

$$A_i = \sum_{j=1}^{n_1} E_j - \sum_{j=n_1+1}^p E_j + \sum_{j=1}^{m_1} F_j - \sum_{j=m_1+1}^q F_j + \sum_{j=1}^{m_2} \beta_j - \sum_{j=m_2+1}^{q^{(1)}} \beta_j + \sum_{j=1}^{m_3} \alpha_j - \sum_{j=n_2+1}^{q^{(2)}} \alpha_j > 0 \quad (6)$$

$$B_i = \sum_{j=1}^{n_1} E'_j - \sum_{j=n_1+1}^p E_j + \sum_{j=1}^{m_1} F'_j - \sum_{j=m_1+1}^q F'_j + \sum_{j=1}^{m_3} \delta_j - \sum_{j=m_3+1}^{q_i^{(2)}} \delta_j + \sum_{j=1}^{n_3} \gamma_j - \sum_{j=n_3+1}^{p_i^{(2)}} \gamma_j > 0 \quad (7)$$

where $m_1, n_1; m_2, n_2; m_3, n_3$ and $p, q; p_i^{(1)}, q_i^{(1)}; p_i^{(2)}, q_i^{(2)}$ are non zero integers satisfying the conditions.

$0 \leq n_1 \leq p, 0 \leq n_2 \leq p_i^{(1)}, 0 \leq n_3 \leq p_i^{(2)}, 0 \leq m_1 \leq q, 0 \leq m_2 \leq q_i^{(1)}, 0 \leq m_3 \leq q_i^{(2)}$ you all $i = 1, 2, \dots, r$, where r is positive integers, for asymptotic expansion and all other conditions, (see [1]). These condition will be assumed two hold good in the I-function of two variables occurring in this paper.

The generalized M-series is defined as:

$${}_p M_q^{\alpha, \beta}(z) = {}_p M_1^{\alpha, \beta}(a_1, \dots, a_p; b_1, \dots, b_q; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k \dots (a_p)_k}{(b_1)_k \dots (b_q)_k} \cdot \frac{z^k}{\Gamma(\alpha k + \beta)} \quad (8)$$

where $\alpha, \beta \in \mathbb{C}, z \in \mathbb{C}, \operatorname{Re}(\alpha) > 0; (q_i)_k (i = 1, \dots, p)$ and $(b_j)_k (j = 1, \dots, q)$ are Pochhammer symbols. The series (8) is defined when none of the parameters $(b_j)_k$ is a negative integer or zero, then series terminates to a polynomial in z . The series (8) is convergent for all z if $p \leq q$; it is convergent for $|z| < \delta = \alpha^\alpha$ if $p = q+1$ and divergent if $p > q+1$. When $p = q+1$ and $|z| < \delta$, the series is convergent on conditions depending on the parameters. The detailed account of the M-series are given in [2].

II. THEOREM-1

$$\begin{aligned} \int_0^1 z^{p-1} (t-z)^{\sigma-1} {}_p M_q^{\alpha, \beta} [bz^\nu (t-z)^\tau] I_{p, q; Q; r}^{m, n, p} \left[\begin{matrix} \omega_1 z^{\nu_1} (t-z)^{\tau_1} \\ \omega_2 z^{\nu_2} (t-z)^{\tau_2} \end{matrix} \middle| \begin{matrix} ((P)), ((Q)), ((R)), ((S)), ((T)) \\ ((U)), ((V)), ((W)), ((X)), ((Y)) \end{matrix} \right] dz \\ = t^{\sigma+p-z} \sum_{k=0}^{\infty} \frac{(a_1)_k \dots (a_p)_k}{(b_1)_k \dots (b_q)_k} \cdot \frac{[bt^{\nu+\tau}]^k}{\Gamma(\alpha k + \beta)} \\ I_{p_1+2, q_1+1; Q; r}^{m, n+2; p} \left[\begin{matrix} \omega_1 t^{(\nu_1+\tau).z} \\ \omega_2 t^{(\nu_2+\tau).z} \end{matrix} \middle| \begin{matrix} ((X_1)), ((P)), ((Q)), ((S)), ((T)) \\ ((U)), ((X_2)), ((V)), ((W)), ((X)), ((Y)) \end{matrix} \right] \end{aligned}$$

where X_1 and X_2 are as follows:

$$X_1 \equiv \{(+1-p-\nu k : \nu_1, \nu_2), (+1-\sigma-\tau k : \tau_1, \tau_2)\}$$

$$X_2 \equiv \{(+1 - \sigma - p - vk - \tau k) : (\nu_1 + \tau_1), (\nu_2 + \tau_2)\}$$

provided that,

- (i) $\alpha > 0, \beta > 0,$
- (ii) $\operatorname{Re} \left[p + \tau_1 \left[\min_{1 \leq j < m_2} \left(\frac{b_j}{\beta_j} \right) \right] + \tau_2 \left[\min_{1 \leq j < m_3} \left(\frac{c_j}{\gamma_j} \right) \right] \right] > 0,$
- (iii) $\operatorname{Re} \left[p + \nu_1 \left[\min_{1 \leq j < m_2} \left(\frac{b_j}{\beta_j} \right) \right] + \nu_2 \left[\min_{1 \leq j < m_3} \left(\frac{c_j}{\gamma_j} \right) \right] \right] > 0,$
- (iv) $\Delta_1 < \frac{1}{2} \pi A_i, \Delta_2 < \frac{1}{2} \pi B_i, \forall i \in (1, 2, \dots, r),$

(9)

where A_i and B_i are mentioned in (6) and (7) respectively.

III. THEOREM-2

$$\begin{aligned} & \int_0^1 z^{p-1} (t-z)^{\sigma-1} {}_p M_q^{\alpha, \beta} [bz^{\nu} (t-z)^{\tau}] \cdot I_{p, q; Q; r}^{m, n, p} \left[\begin{matrix} \omega_1 z^{\nu_1} (t-z)^{\tau_1} \\ \omega_2 z^{\nu_2} (t-z)^{\tau_2} \end{matrix} \middle| \begin{matrix} ((P)), ((Q)), ((R)), ((S)), ((T)) \\ ((U)), ((V)), ((W)), ((X)), ((Y)) \end{matrix} \right] dz \\ &= t^{\sigma+p-z} \sum_{k=0}^{\infty} \frac{(a_1)_k \dots (a_p)_k}{(b_1)_k \dots (b_q)_k} \frac{[bt^{\nu+\tau}]^k}{\Gamma(\alpha k + \beta)} \\ & I_{p_1+2, q_1+1; Q; r}^{m, n+2; p} \left[\begin{matrix} \omega_1 t^{(\nu_1+\tau)_z} \\ \omega_2 t^{(\nu_2+\tau)_z} \end{matrix} \middle| \begin{matrix} ((X_3)), ((P)), ((Q)), ((S)), ((T)) \\ ((U)), ((X_4)), ((V)), ((W)), ((X)), ((Y)) \end{matrix} \right] \end{aligned}$$

where X_1 and X_2 are as follows:

$$\begin{aligned} X_1 &\equiv \{(+1 - p - vk : \nu_1, \nu_2), (+1 - \sigma - \tau k : \tau_1, \tau_2)\} \\ X_2 &\equiv \{(+1 - \sigma - p - vk - \tau k) : (\nu_1 + \tau_1), (\nu_2 + \tau_2)\} \end{aligned}$$

provided that:

- (i) $\alpha > 0, \beta > 0,$
- (ii) $\operatorname{Re} \left[p + \tau_1 \left[\min_{1 \leq j < m_2} \left(\frac{b_j}{\beta_j} \right) \right] + \tau_2 \left[\min_{1 \leq j < m_3} \left(\frac{c_j}{\gamma_j} \right) \right] \right] > 0,$
- (iii) $\operatorname{Re} \left[p + \nu_1 \left[\min_{1 \leq j < m_2} \left(\frac{b_j}{\beta_j} \right) \right] + \nu_2 \left[\min_{1 \leq j < m_3} \left(\frac{c_j}{\gamma_j} \right) \right] \right] > 0,$
- (iv) $\Delta_1 < \frac{1}{2} \pi A_i, \Delta_2 < \frac{1}{2} \pi B_i, \forall i \in (1, 2, \dots, r),$

(10)

where A_i and B_i are mentioned in (6) and (7) respectively.

IV. HEOREM-3

$$\int_0^1 z^{p-1} (t-z)^{\sigma-1} {}_p M_q^{\alpha, \beta} [bz^{\nu} (t-z)^{\tau}] \cdot I_{p, q; Q; r}^{m, n, p} \left[\begin{matrix} \omega_1 z^{-\nu_1} (t-z)^{\tau_1} \\ \omega_2 z^{-\nu_2} (t-z)^{\tau_2} \end{matrix} \middle| \begin{matrix} ((P)), ((Q)), ((R)), ((S)), ((T)) \\ ((U)), ((V)), ((W)), ((X)), ((Y)) \end{matrix} \right] dz$$

$$= t^{\sigma+p-z} \sum_{k=0}^{\infty} \frac{(a_1)_k \dots (a_p)_k}{(b_1)_k \dots (b_q)_k} \frac{[bt^{(\nu+\tau)}]^k}{\Gamma(\alpha k + \beta)}.$$

$$I_{p_1+1, n+1; p}^{m+1, n+1; p} \left[\begin{matrix} \omega_1 t^{(\tau_1-\nu_1)z} \\ \omega_2 t^{(\tau_2-\nu_2)z} \end{matrix} \middle| \begin{matrix} ((X_5)), ((P)), ((Q)), ((S)), ((T)) \\ ((U)), ((X_6)), ((V)), ((W)), ((X)), ((Y)) \end{matrix} \right]$$

where $((P)), ((Q)), ((R)), ((S)), ((T)), ((U)), ((V)), ((W)), ((X)), ((Y))$ are sets of parameters given in (1) and $((X_5)), ((X_6))$ are as follows:

$$X_5 \equiv \{(p - \nu k : \nu_1, \nu_2), (1 - \sigma - \tau k : \tau_1, \tau_2)\}$$

$$X_6 \equiv \{(1 - p - \sigma - \nu k - \tau k) : (\tau_1 - \nu_1), (\tau_2 - \nu_2)\}$$

provided that

- (i) $\alpha > 0, \beta > 0,$
- (ii) $\operatorname{Re} \left[\sigma + \tau_1 \left[\min_{1 \leq j \leq m_2} \left(\frac{b_j}{\beta_j} \right) \right] + \tau_2 \left[\min_{1 \leq j \leq m_3} \left(\frac{c_j}{\gamma_j} \right) \right] \right] > 0,$
- (iii) $\operatorname{Re} \left[p + \nu_1 \left[\min_{1 \leq j \leq m_2} \left(\frac{b_j}{\beta_j} \right) \right] + \nu_2 \left[\min_{1 \leq j \leq m_3} \left(\frac{c_j}{\gamma_j} \right) \right] \right] > 0,$
- (iv) $\Delta_1 < \frac{1}{2} \pi A_i, \Delta_2 < \frac{1}{2} \pi B_i, \forall i \in (1, 2, \dots, r),$

where A_i and B_i are mentioned in (6) and (7) respectively.

V. THEOREM-4

$$\int_0^1 z^{p-1} (t-z)^{\sigma-1} {}_pM_q^{\alpha, \beta} [bz^{\nu} (t-z)^{\tau}] \cdot I_{p, q; Q; r}^{m, n, p} \left[\begin{matrix} \omega_1 z^{-\nu_1} (t-z)^{-\tau_1} \\ \omega_2 z^{-\nu_2} (t-z)^{-\tau_2} \end{matrix} \middle| \begin{matrix} ((P)), ((Q)), ((R)), ((S)), ((T)) \\ ((U)), ((V)), ((W)), ((X)), ((Y)) \end{matrix} \right] dz$$

$$= t^{\sigma+p-2} \sum_{k=0}^{\infty} \frac{(a_1)_k \dots (a_p)_k}{(b_1)_k \dots (b_q)_k} \frac{[bt^{(\nu+\tau)}]^k}{\Gamma(\alpha k + \beta)}.$$

$$I_{p_1+2, q_1+1; Q; r}^{m+1, n+2} \left[\begin{matrix} \omega_1 t^{-(\nu_1+\tau_1)z} \\ \omega_2 t^{-(\nu_2+\tau_2)z} \end{matrix} \middle| \begin{matrix} ((X_7)), ((P)), ((Q)), ((S)), ((T)) \\ ((U)), ((X_8)), ((V)), ((W)), ((X)), ((Y)) \end{matrix} \right]$$

where $((P)), ((Q)), ((R)), ((S)), ((T)), ((U)), ((V)), ((W)), ((X)), ((Y))$ are sets of parameters given in (1) and $((X_7)), ((X_8))$ are as follows:

$$X_7 \equiv \{(p + \nu k : \nu_1, \nu_2), (\sigma + \tau k : \tau_1, \tau_2)\},$$

$$X_8 \equiv \{p + \sigma + (\nu + \tau)k; (\nu_1 + \nu_2), (\tau_1 + \tau_2)\},$$

provided that:

- (i) $\alpha > 0, \beta > 0,$
- (ii) $\operatorname{Re} \left[\sigma + \tau_1 \left[\min_{1 \leq j \leq m_2} \left(\frac{b_j}{\beta_j} \right) \right] + \tau_2 \left[\min_{1 \leq j \leq m_3} \left(\frac{c_j}{\gamma_j} \right) \right] \right] > 0,$
- (iii) $\operatorname{Re} \left[p + \nu_1 \left[\min_{1 \leq j \leq m_2} \left(\frac{b_j}{\beta_j} \right) \right] + \nu_2 \left[\min_{1 \leq j \leq m_3} \left(\frac{c_j}{\gamma_j} \right) \right] \right] > 0,$

$$(iv) \quad \Delta_1 < \frac{1}{2} \pi A_i, \Delta_2 < \frac{1}{2} \pi B_i, \forall i \in (1, 2, \dots, r), \quad (12)$$

where A_i and B_i are mentioned in (6) and (7) respectively.

VI. PROOF

If we apply the definition of I-function of two variables and M-series to the given (9) integral, we have:

$$\int_0^t z^{p-1} (t-z)^{\sigma-1} {}_pM_q^{\alpha, \beta} [bz^\nu (t-z)^\tau] \cdot I_{p, q; Q; \tau}^{m, n, p} \left[\begin{matrix} \omega_1 z^{\nu_1} (\tau-z)^{\tau_1} \\ \omega_2 z^{\nu_2} (\tau-z)^{\tau_2} \end{matrix} \middle| \begin{matrix} ((P)), ((Q)), ((R)), ((S)), ((T)) \\ ((U)), ((V)), ((W)), ((X)), ((Y)) \end{matrix} \right] dz$$

$$\int_0^t z^{p-1} (t-z)^{\sigma-1} \sum_{k=0}^{\infty} \frac{(a_1)_k \dots (a_p)_k}{(b_1)_k \dots (b_q)_k} \cdot \frac{b^k z^{\nu k} (t-z)^{\tau k}}{\Gamma(\alpha k + \beta)} \cdot$$

$$\times \frac{1}{(2\pi\omega)^2} \int_{L_1} \int_{L_2} \phi_1(\xi) \cdot \phi_2(\eta) \cdot \psi(\xi, \eta) \omega_1^\xi z^{\nu_1 \xi} (t-z)^{\tau_1 \xi} \cdot \omega_2^\eta z^{\nu_2 \eta} (t-z)^{\tau_2 \eta} d\xi \cdot d\eta \cdot dz$$

Interchanging the order of integration and summation which is permissible under the condition stated with theorem, we get:

$$\sum_{k=0}^{\infty} \frac{(a_1)_k \dots (a_p)_k}{(b_1)_k \dots (b_q)_k} \cdot \frac{b^k t^{(\nu \tau)}}{\Gamma(\alpha k + \beta)} \cdot \int_0^t z^{p+\nu k + \nu_1 \xi + \nu_2 \eta} \cdot (t-z)^{\sigma + \tau k + \tau_1 \xi + \tau_2 \eta - 1}$$

$$\times \frac{1}{(2\pi\omega)^2} \int_{L_1} \int_{L_2} \phi_1(\xi) \cdot \phi_2(\eta) \cdot \psi(\xi, \eta) \omega_1^\xi d\xi \cdot \omega_2^\eta d\eta \cdot dz$$

Substituting $z = st$ and, using the formula of Beta function and after adjusting the terms we get theorem (9).

VII. APPLICATIONS

(i) If we set $\nu_1 = 0$ and $\nu_2 = 0$ in (9) then we get,

$$\int_0^t z^{p-1} (t-z)^{\sigma-1} {}_pM_q^{\alpha, \beta} [bz^\nu (t-z)^\tau] \cdot I_{p, q; Q; \tau}^{m, n, p} \left[\begin{matrix} \omega_1 (\tau-z)^{\tau_1} \\ \omega_2 (\tau-z)^{\tau_2} \end{matrix} \middle| \begin{matrix} ((P)), ((Q)), ((R)), ((S)), ((T)) \\ ((U)), ((V)), ((W)), ((X)), ((Y)) \end{matrix} \right] dz$$

$$t^{\sigma+p-2} \cdot \sum_{k=0}^{\infty} \frac{(a_1)_k \dots (a_p)_k}{(b_1)_k \dots (b_q)_k} \cdot \frac{(bt^{\nu+\tau})^k}{\Gamma(\alpha k + \beta)} \cdot \Gamma(p + \nu k) \cdot$$

$$I_{p+1, q+1; Q; \tau}^{m, n, p} \left[\begin{matrix} \omega_1 t^{\tau_1} z \\ \omega_2 t^{\tau_2} z \end{matrix} \middle| \begin{matrix} ((X_9)), ((P)), ((Q)), ((R)), ((S)), ((T)) \\ ((U)), ((X_{10})), ((V)), ((Q)), ((X)), ((Y)) \end{matrix} \right]$$

where,

$$X_9 \equiv \{(1 - \sigma - \tau k : \tau_1, \tau_2)\},$$

$$X_{10} \equiv \{(1 - \sigma - p - \nu k - \tau k : \tau_1, \tau_2)\},$$

provided that:

(1) $\alpha, \beta > 0$,

$$(2) \quad \operatorname{Re} \left[\sigma + \tau_1 \left[\min_{1 \leq j < m_2} \left(\frac{b_j}{\beta_j} \right) \right] + \tau_2 \left[\min_{1 \leq j < m_3} \left(\frac{c_j}{\gamma_j} \right) \right] \right] > 0,$$

$$(3) \quad \Delta_1 < \frac{1}{2} \pi A_i, \Delta_2 < \frac{1}{2} \pi B_i, \forall i \in (1, 2, \dots, r), \quad (13)$$

where A_i and B_i are mentioned in (6) and (7) respectively.

(ii) If we set $\tau_1 = 0$ and $\tau_2 = 0$ in (9) then we get:

$$\int_0^1 z^{p-1} (t-z)^{\sigma-1} {}_p M_q^{\alpha, \beta} [bz^\nu (t-z)^\tau] \cdot I_{p, q; Q; \tau}^{m, n, p} \left[\begin{matrix} \omega_1 z^{\nu_1} \\ \omega_2 z^{\nu_2} \end{matrix} \middle| \begin{matrix} ((P)), ((Q)), ((R)), ((S)), ((T)) \\ ((U)), ((V)), ((W)), ((X)), ((Y)) \end{matrix} \right]$$

$$t^{\sigma+p-2} \cdot \sum_{k=0}^{\infty} \frac{(a_1)_k \dots (a_p)_k}{(b_1)_k \dots (b_q)_k} \cdot \frac{(bt^{\nu+\tau})^k}{\Gamma(\alpha k + \beta)} \cdot \Gamma(\sigma + \tau k).$$

$$I_{p+1, q+1; Q; \tau}^{m, n+1, p} \left[\begin{matrix} \omega_1 t^{\nu_1} z \\ \omega_2 t^{\nu_2} z \end{matrix} \middle| \begin{matrix} ((x_{11})), ((P)), ((Q)), ((R)), ((S)), ((T)) \\ ((U)), ((x_{12})), ((V)), ((W)), ((X)), ((Y)) \end{matrix} \right]$$

where,

$$X_{11} \equiv \{(1-p-\nu k) : \nu_1, \nu_2\},$$

$$X_{12} \equiv \{(1-p-\sigma-\nu k-\tau k) : \nu_1, \nu_2\},$$

provided that

$$(1) \quad \alpha, \beta > 0,$$

$$(2) \quad \operatorname{Re} \left\{ p + \nu_1 \left[\min_{1 \leq j < m_2} \left(\frac{b_j}{\beta_j} \right) \right] + \nu_2 \left[\min_{1 \leq j < m_3} \left(\frac{c_j}{\gamma_j} \right) \right] \right\} > 0,$$

$$(3) \quad \Delta_1 < \frac{1}{2} \pi A_i, \Delta_2 < \frac{1}{2} \pi B_i, \quad (14)$$

where A_i and B_i are mentioned in (6) and (7) respectively.

CONFLICT OF INTEREST

Authors declare that they do not have any conflict of interest.

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