European Journal of Mathematics and Statistics
Vol 6 | Issue 5 | September 2025
ISSN 2736-5484

RESEARCH ARTICLE

On Legendre Cordial Labeling of Some Graphs

Jason D. Andoyo and Ricky F. Rulete*

ABSTRACT

For a simple connected graph $G = (V, E)$, a bijective function $f: V(G) \to \{1, 2, \ldots, |V|\}$ is said to be a Legendre cordial labeling modulo p, where p is an odd prime, if the induced function $f_p^*: E(G) \to \{0, 1\}$ defined by $f_p^*(uv) = 0$ if $(\omega_{uv}/p) = -1$ or $\omega_{uv} = 0$, and $f_p^*(uv) = 1$ if $(\omega_{uv}/p) = 1$, where $f(u) + f(v) \equiv \omega_{uv} (mod\, p)$, $0 \le \omega_{uv} < p$, satisfies the condition $\left| e_{f_p^*}(0) - e_{f_p^*}(1) \right| \le 1$, where $e_{f_p^*}(i)$ is the number of edges with label i $(i = 0, 1)$. In this paper, we determined the values of n, m, and k so that the following special graphs admit a Legendre cordial labeling modulo p for an odd prime p: path graph P_n, cycle graph C_n, star graph S_n, tadpole graph $T_{n,m}$, kayak paddle graph $KP_{n,m,k}$, bistar graph $B_{n,m}$, and fan graph F_n.

Keywords: Legendre cordial labeling, legendre symbol, modulo, odd prime.

Submitted: August 07, 2025
Published: September 08, 2025

10.24018/ejmath.2025.6.5.408

University of Southeastern Philippines, Davao City, Philippines.

***Corresponding Author*
e-mail: ricky.rulete@usep.edu.ph

1. Introduction

In this study, only graphs that were finite, simple, and connected were considered. A graph $G = (V, E)$ has a vertex set $V = V(G)$ and an edge set $E = E(G)$, and the cardinalities $|V|$ and $|E|$ are called the order and size of G, respectively. For a general reference to graph-theoretic notions, refer to [1]. Graph labeling is a concept in graph theory in which integers are assigned to vertices or edges (or both) subject to certain conditions. This topic is an active area of research with numerous applications in coding theory and communications.

In the number theory, finding solutions to certain equations or congruences (subject to certain conditions) is very important. Quadratic congruence $x^2 \equiv a(\text{mod } p)$, where p is an odd prime, has been studied for many years, and several properties have been established to test whether the congruence has a solution. Legendre, a French mathematician, introduced the Legendre symbol (a/p) which states that if p is an odd prime and a is an integer relatively prime to p, then $(a/p) = 1$ whenever $x^2 \equiv a(\text{mod } p)$ has a solution; otherwise, $(a/p) = -1$. Several properties of the Legendre symbol can be found in previous studies [2], [3]. Various topics in number theory such as encryption, decryption, and the Fibonacci series, are now significant in the fields of cryptography, computer science, and engineering [4].

In 1987, Cahit [5] introduced a new graph-labeling method called cordial labeling. This graph labeling is considered a weaker version of graceful labeling and harmonious labeling. It also serves as a foundation for different topics of graph labeling, such as total product cordial labeling [6], Lucas divisor cordial labeling [7], and Fibonacci cordial labeling [8]. In fact, some topics of cordial labeling, namely, sum cordial labeling and mean cordial labeling, are utilized to study blood circulation, human anatomy, and the circulatory system [9], [10].

In this study, we introduce a new type of cordial labeling, called Legendre cordial labeling. Let p be an odd prime. For a simple connected graph $G = (V, E)$ of order n, a bijective function $f: V(G) \to \{1, 2, \ldots, n\}$ is said to be a Legendre cordial labeling modulo p if the induced function $f_p^*: E(G) \to \{0, 1\}$ is defined by

$$f_p^*(uv) = \begin{cases} 0 \text{ if } (\omega_{uv}/p) = -1 \text{ or } \omega_{uv} = 0 \\ 1 \text{ if } (\omega_{uv}/p) = 1 \end{cases}$$

given that $f(u)+f(v) \equiv \omega_{uv} (\text{mod}\, p)$, $0 \le \omega_{uv} < p$, satisfies the condition $\left|e_{f_p^*}(0) - e_{f_p^*}(1)\right| \le 1$, where $e_{f_p^*}(i)$ is the number of edges with label i $(i = 0, 1)$. A graph that admits a Legendre cordial labeling modulo p is called a Legendre cordial graph modulo p.

Additionally, this new type of cordial labeling extends the traditional graph labeling methods by incorporating the Legendre symbol. In addition, this approach aims to investigate new mathematical structures in graph labeling and to explore their potential applications in real life.

2. Preliminaries

Definition 2.1. A *path* graph P_n of order n is a graph with vertex set $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(P_n) = \{v_1v_2, v_2v_3, \ldots, v_{n-1}v_n\}$.

Definition 2.2. A *cycle* graph C_n of order n is a graph with vertex set $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(C_n) = \{v_1v_2, v_2v_3, \ldots, v_{n-1}v_n, v_nv_1\}$.

Definition 2.3. A *star* graph S_n of order n is a graph obtained from vertices $v_1, v_2, \ldots, v_{n-1}$ and connects each of these vertices to a central vertex v_n.

Definition 2.4. A *tadpole* graph $T_{n,m}$ of order $n+m$ is obtained from a cycle graph C_n and path graph P_{m+1} where a pendant vertex (a vertex with degree 1) is a vertex of C_n.

Definition 2.5. A *kayak paddle* graph $KP_{n,m,k}$ of order $n+m+k$ is a graph obtained from two cycle graphs C_n and C_k, and path graph P_{m+2} for which a pendant vertex of P_{m+2} is a vertex of C_n and the other pendant vertex is a vertex of C_k.

Definition 2.6. A *bistar* graph $B_{n,m}$ of order $n+m$ is obtained from two star graphs S_n and S_m such that the central vertex of S_n is adjacent to the central vertex of S_m.

Definition 2.7. A fan graph F_n of order n is obtained from path graph P_{n-1} and connects each vertex of P_{n-1} to a new vertex x_0.

Definition 2.8. Let p be an odd prime and a be an integer relatively prime to p. Then a is a *quadratic residue* of p if the quadratic congruence $x^2 \equiv a(mod\ p)$ has a solution. Otherwise, a is a *quadratic nonresidue* of p. The *Legendre symbol* (a/p) is defined as $(a/p) = 1$ if a is a quadratic residue of p; otherwise, $(a/p) = -1$.

Theorem 2.9. [3] *Let p be an odd prime. Then p has $\frac{p-1}{2}$ quadratic residues and $\frac{p-1}{2}$ quadratic nonresidues in the set* $\{1, 2, \ldots, p-1\}$.

Theorem 2.10. [3] *Let p be an odd prime and let a and b be integers relatively prime to p. If* $a \equiv b (\text{mod}\, p)$, *then* $(a/p) = (b/p)$.

3. Main Results

In the succeeding discussion, we denote p as an odd prime.

Lemma 3.1. *There exists an integer k,* $0 \le k < p$, *such that* $\left(\frac{p+2k\pm1}{2}/p\right) = 1$.

Proof. Define $\theta_k \equiv (p+2k\pm1)/2 (\text{mod}\, p)$, $0 \le \theta_k < p$, for $k = 1, 2, \ldots, p-1$. Let $\zeta = \{\theta_k : k = 0, 1, \ldots, p-1\}$, then $\zeta = \{0, 1, \ldots, p-1\}$. According to Theorem 2.9, p has $\frac{p-1}{2}$ quadratic residues in set ζ. Thus, $(\theta_k/p) = 1$ for some integer k, $0 \le k < p$. Because $\left(\frac{p+2k\pm1}{2} / p\right) = (\theta_k/p)$ by Theorem 2.10, there exists an integer k, $0 \le k < p$, such that $\left(\frac{p+2k\pm1}{2} / p\right) = 1$. □

Theorem 3.2. *The path graph P_p is a Legendre cordial graph modulo p.*

Proof. Let $V(P_p) = \{v_1, v_2, \ldots, v_p\}$ and define the bijective function $f\colon V(P_p) \to \{1, 2, \ldots, p\}$ by

$$f(v_i) = \begin{cases} i + \dfrac{p-1}{2} & \text{for } i = 1, 2, \ldots, \dfrac{p+1}{2} \\ i - \dfrac{p+1}{2} & \text{for } i = \dfrac{p+3}{2}, \dfrac{p+5}{2}, \ldots, p. \end{cases}$$

As a consequence, for $v_iv_{i+1} \in E(P_p)$,

$$f(v_i) + f(v_{i+1}) \equiv \begin{cases} 2i \,(\text{mod}\, p) & \text{for } i = 1, 2, \ldots, \dfrac{p-1}{2} \\ (2i-p)\,(\text{mod}\, p) & \text{for } i = \dfrac{p+1}{2}, \dfrac{p+3}{2}, \ldots, p-1. \end{cases}$$

In view of this labeling, let

$$\alpha = \{\xi : f(v_i) + f(v_{i+1}) \equiv \xi (\text{mod}\, p), 0 < \xi < p, i = 1, 2, \ldots, p-1\}.$$

Thus, $\alpha = \{1, 2, \ldots, p-1\}$ and by Theorem 2.9, p has $\frac{p-1}{2}$ quadratic residues and $\frac{p-1}{2}$ quadratic nonresidues in set α. Hence, $e_{f_p^*}(0) = e_{f_p^*}(1) = \frac{p-1}{2}$. Therefore, $\left|e_{f_p^*}(0) - e_{f_p^*}(1)\right| = 0$ and it follows that P_p admits a Legendre cordial labeling modulo p. □

Example 3.3. Fig. 1 presents the Legendre cordial labeling modulo p of the path graph P_p, specifically illustrated for the case $p = 7$.

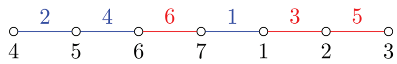

Fig. 1. Legendre cordial labeling modulo 7 of P_7.

Theorem 3.4. *The cycle graph C_p is a Legendre cordial graph modulo p.*

Proof. Let $V(C_p) = \{v_1, v_2, \ldots, v_p\}$ and suppose that $f\colon V(C_p) \to \{1, 2, \ldots, p\}$ is a bijective function defined by $f(v_i) = i$ for $i = 1, 2, \ldots, p$. Consequently, for $v_i v_{i+1} \in E(C_p)$,

$$f(v_i) + f(v_{i+1}) \equiv \begin{cases} (2i+1) \pmod p & \text{for } i = 1, 2, \ldots, \dfrac{p-3}{2} \\ 0 \pmod p & \text{for } i = \dfrac{p-1}{2} \\ (2i-p+1) \pmod p & \text{for } i = \dfrac{p+1}{2}, \dfrac{p+3}{2}, \ldots, p, \end{cases}$$

and for $v_1 v_p \in E(C_p)$, $f(v_1) + f(v_p) \equiv 1 \pmod p$. In view of the above labeling, suppose that

$$\alpha = \{\xi : f(v_i) + f(v_{i+1}) \equiv \xi \pmod p, 0 < \xi < p, i = 1, 2, \ldots, p, i \neq (p-1)/2\}$$

and

$$\beta = \left\{\xi : f(v_1) + f\left(v_p\right) \equiv \xi \pmod p, 0 < \xi < p\right\}.$$

Thus, $\alpha \cup \beta = \{1, 2, \ldots, p-1\}$. By Theorem 2.9, there are $\frac{p-1}{2}$ quadratic residues and $\frac{p-1}{2}$ quadratic nonresidues of p in the set $\alpha \cup \beta$. Additionally, notice that $f_p^*(v_{(p-1)/2} v_{(p+1)/2}) = 0$. Therefore, $e_{f_p^*}(0) = \frac{p-1}{2} + 1$ and $e_{f_p^*}(1) = \frac{p-1}{2}$. Hence, $\left|e_{f_p^*}(0) - e_{f_p^*}(1)\right| = 1$ and it follows that C_p admits a Legendre cordial labeling modulo p. □

Example 3.5. The Legendre cordial labeling modulo p of the cycle graph C_p, with $p = 7$, is shown in Fig. 2.

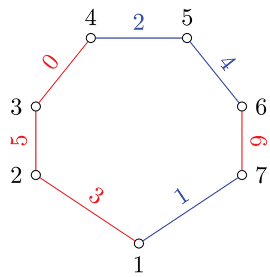

Fig. 2. Legendre cordial labeling modulo 7 of C_7.

Theorem 3.6. *The star graph S_p is a Legendre cordial graph modulo p.*

Proof. Let $V(S_p) = \{v_1, v_2, \ldots, v_p\}$ where v_p is a central vertex. Now, let $f\colon V(S_p) \to \{1, 2, \ldots, p\}$ be a bijective function defined by $f(v_i) = i$ for $i = 1, 2, \ldots, p$. So, for $v_i v_p \in E(S_p)$, $f(v_i) + f(v_p) \equiv i \pmod p$ for $i = 1, 2, \ldots, p-1$. In view of this labeling, suppose that

$$\alpha = \left\{\xi : f(v_i) + f\left(v_p\right) \equiv \xi \pmod p, 0 < \xi < p, i = 1, 2, \ldots, p-1\right\}.$$

Consequently, $\alpha = \{1, 2, \ldots, p-1\}$. According to Theorem 2.9, there are $\frac{p-1}{2}$ quadratic residues and $\frac{p-1}{2}$ quadratic nonresidues of p in set α. Therefore, $e_{f_p^*}(0) = e_{f_p^*}(1) = \frac{p-1}{2}$ and it follows that $\left|e_{f_p^*}(0) - e_{f_p^*}(1)\right| = 0$. Hence, S_p admits a Legendre cordial labeling modulo p. □

Example 3.7. An illustration of the Legendre cordial labeling modulo p of the star graph S_p for $p = 7$ is provided in Fig. 3.

Theorem 3.8. *The tadpole graph $T_{p,p}$ is a Legendre cordial graph modulo p.*

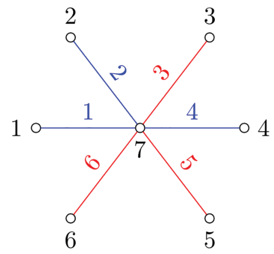

Fig. 3. Legendre cordial labeling modulo 7 of S_7.

Proof. According to Lemma 3.1, there exists an integer k, $0 \le k < p$, such that $\left(\frac{p+2k+1}{2}/p\right) = 1$. Let C_p be a cycle and P_{p+1} be a path of $T_{p,p}$ with $V(C_p) = \{v_1, v_2, \ldots, v_p\}$ and $V(P_{p+1}) = \{v_k, u_1, u_2, \ldots, u_p\}$. We define the bijective function $f\colon V(T_{p,p}) \to \{1, 2, \ldots, 2p\}$ by $f(v_i) = i$ for $i = 1, 2, \ldots, p$, and

$$f(u_i) = \begin{cases} i + \dfrac{p-1}{2} + p & \text{for } i = 1, 2, \ldots, \dfrac{p+1}{2} \\ i - \dfrac{p+1}{2} + p & \text{for } i = \dfrac{p+3}{2}, \dfrac{p+5}{2}, \ldots, p. \end{cases}$$

For the edges of C_p,

$$f(v_i) + f(v_{i+1}) \equiv \begin{cases} (2i+1) \pmod p & \text{for } i = 1, 2, \ldots, \dfrac{p-3}{2} \\ 0 \pmod p & \text{for } i = \dfrac{p-1}{2} \\ (2i - p + 1) \pmod p & \text{for } i = \dfrac{p+1}{2}, \dfrac{p+3}{2}, \ldots, p, \end{cases}$$

and $f(v_1) + f(v_p) \equiv 1 (\bmod p)$. For the edges of P_{p+1},

$$f(u_i) + f(u_{i+1}) \equiv \begin{cases} 2i \pmod p & \text{for } i = 1, 2, \ldots, \dfrac{p-1}{2} \\ (2i - p) \pmod p & \text{for } i = \dfrac{p+1}{2}, \dfrac{p+3}{2}, \ldots, p-1. \end{cases}$$

and $f(v_k) + f(u_1) \equiv \frac{p+2k+1}{2} (\bmod p)$.

In view of the above labeling, let

$$\alpha = \{\xi : f(v_i) + f(v_{i+1}) \equiv \xi (\bmod p), 0 < \xi < p, i = 1, 2, \ldots, p, i \ne (p-1)/2\},$$

$$\beta = \{\xi : f(v_1) + f(v_p) \equiv \xi (\bmod p), 0 < \xi < p\}, \textit{and}$$

$$\gamma = \{\xi : f(u_i) + f(u_{i+1}) \equiv \xi (\bmod p), 0 < \xi < p, i = 1, 2, \ldots, p-1\}.$$

Thus, $\alpha \cup \beta = \gamma = \{1, 2, \ldots, p-1\}$. According to Theorem 2.9, there are $\frac{p-1}{2}$ quadratic residues and $\frac{p-1}{2}$ quadratic nonresidues of p in the sets $\alpha \cup \beta$ and γ. Additionally, notice that $f_p^*(v_{(p-1)/2} v_{(p+1)/2}) = 0$ and $f_p^*(v_k u_1) = 1$. Therefore, $e_{f_p^*}(0) = e_{f_p^*}(1) = 2\left(\frac{p-1}{2}\right) + 1 = p$. Hence, $\left|e_{f_p^*}(0) - e_{f_p^*}(1)\right| = 0$ and it follows that $T_{p,p}$ admits a Legendre cordial labeling modulo p. □

Example 3.9. In Fig. 4, one can observe the Legendre cordial labeling modulo p of the tadpole graph $T_{p,p}$ for a particular case when $p = 7$.

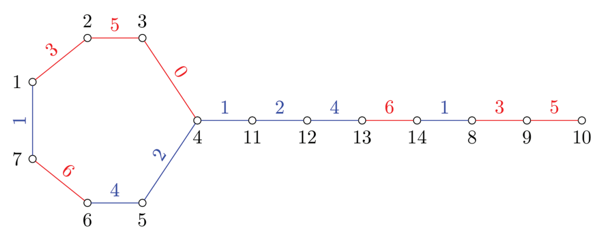

Fig. 4. Legendre cordial labeling modulo 7 of $T_{7,7}$.

Theorem 3.10. *The kayak paddle graph $KP_{p,p,p}$ is a Legendre cordial graph modulo p.*

Proof. Note that there exist integers k_1 and k_2, $0 \leq k_1, k_2 < p$, for which $\left(\frac{p+2k_1+1}{2} / p\right) = \left(\frac{p+2k_2-1}{2}/p\right) = 1$ according to Lemma 3.1. Now, suppose that C_p^1 and C_p^2 are cycles of $KP_{p,p,p}$ with $V(C_p^j) = \{v_1^j, v_2^j, \ldots, v_p^j\}$ for $j = 1,2$, and let P_{p+2} be the path of $KP_{p,p,p}$ with $V(P_{p+2}) = \{v_{k_1}^1, u_1, u_2, \ldots, u_p, v_{k_2}^2\}$. Furthermore, define the bijective function $f\colon V(KP_{p,p,p}) \to \{1,2,\ldots,3p\}$ by $f(v_i^1) = i$ and $f(v_i^2) = i + 2p$ for $i = 1,2,\ldots,p$, and

$$f(u_i) = \begin{cases} i + \dfrac{p-1}{2} + p & \text{for } i = 1,2,\ldots,\dfrac{p+1}{2} \\ i - \dfrac{p+1}{2} + p & \text{for } i = \dfrac{p+3}{2}, \dfrac{p+5}{2}, \ldots, p. \end{cases}$$

For the edges of C_p^j,

$$f\left(v_i^j\right) + f\left(v_{i+1}^j\right) \equiv \begin{cases} (2i+1) \pmod p & \text{for } i = 1,2,\ldots,\dfrac{p-3}{2} \\ 0 \pmod p & \text{for } i = \dfrac{p-1}{2} \\ (2i-p+1) \pmod p & \text{for } i = \dfrac{p+1}{2}, \dfrac{p+3}{2}, \ldots, p, \end{cases}$$

and $f(v_1^j) + f(v_p^j) \equiv 1 \pmod p$ for $j = 1,2$. For the edges of P_{p+2},

$$f(u_i) + f(u_{i+1}) \equiv \begin{cases} 2i \pmod p & \text{for } i = 1,2,\ldots,\dfrac{p-1}{2} \\ (2i-p) \pmod p & \text{for } i = \dfrac{p+1}{2}, \dfrac{p+3}{2}, \ldots, p-1, \end{cases}$$

$f(v_{k_1}^1) + f(u_1) \equiv \dfrac{p+2k_1+1}{2} \pmod p$, and $f(u_p) + f(v_{k_2}^2) \equiv \dfrac{p+2k_2-1}{2} \pmod p$.

In view of the above labeling, let

$$\alpha_j = \{\xi : f(v_i^j) + f(v_{i+1}^j) \equiv \xi \pmod p, 0 < \xi < p, i = 1,2,\ldots,p, i \neq (p-1)/2\}, \textit{and}$$

$$\beta_j = \{\xi : f(v_1^j) + f(v_p^j) \equiv \xi \pmod p, 0 < \xi < p\}$$

for $j = 1,2$, and suppose that

$$\gamma = \{\xi : f(u_i) + f(u_{i+1}) \equiv \xi \pmod p, 0 < \xi < p, i = 1,2,\ldots,p-1\}.$$

So, $\alpha_1 \cup \beta_1 = \alpha_2 \cup \beta_2 = \gamma = \{1,2,\ldots,p-1\}$. By Theorem 2.9, there are $\frac{p-1}{2}$ quadratic residues and $\frac{p-1}{2}$ quadratic nonresidues of p in the sets $\alpha_1 \cup \beta_1$, $\alpha_2 \cup \beta_2$, and γ. Additionally, observe that $f_p^*(v_{(p-1)/2}^j v_{(p+1)/2}^j) = 0$ for $j = 1,2$, and $f_p^*(v_{k_1}^1 u_1) = f_p^*(u_p v_{k_2}^2) = 1$. Thus, $e_{f_p^*}(0) = e_{f_p^*}(1) = 3\left(\frac{p-1}{2}\right) + 2$. Consequently, $\left|e_{f_p^*}(0) - e_{f_p^*}(1)\right| = 0$ and it follows that $KP_{p,p,p}$ admits a Legendre cordial labeling modulo p. □

Example 3.11. Fig. 5 shows the Legendre cordial labeling modulo p of the kayak paddle graph $KP_{p,p,p}$ when $p = 5$.

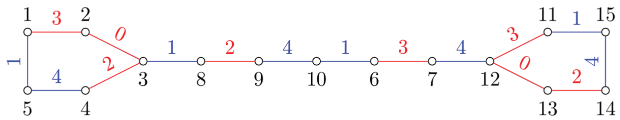

Fig. 5. Legendre cordial labeling modulo 5 of $KP_{5,5,5}$.

Theorem 3.12. *The bistar graph $B_{p,p}$ is a Legendre cordial graph modulo p.*

Proof. Let S_p^1 and S_p^2 be star graphs of $B_{p,p}$ with $V(S_p^j) = \{v_1^j, v_2^j, \ldots, v_p^j\}$ where v_p^j is the central vertex of S_p^j, for $j = 1,2$. Now, let $f\colon V(B_{p,p}) \to \{1,2,\ldots,2p\}$ be a bijective function defined by $f(v_i^j) = i + (j-1)p$ for $i = 1,2,\ldots,p$ and $j = 1,2$. So, for $v_i^j v_p^j \in E\left(B_{p,p}\right)$, $f(v_i^j) + f(v_p^j) \equiv i \pmod p$ for

$i = 1, 2, \ldots, p-1$ and $j = 1, 2$, and for $v_p^1 v_p^2 \in E(B_{p,p}), f(v_p^1) + f(v_p^2) \equiv 0 (\bmod p)$. Given this labeling, let

$$\alpha_j = \left\{\xi : f\left(v_i^j\right) + f\left(v_p^j\right) \equiv \xi \ (\bmod p)\,, 0 < \xi < p, i = 1, 2, \ldots, p-1\right\}$$

for $j = 1, 2$. Thus, $\alpha_1 = \alpha_2 = \{1, 2, \ldots p-1\}$ and by Theorem 2.9, there are $\frac{p-1}{2}$ quadratic residues and $\frac{p-1}{2}$ quadratic nonresidues of p in the sets α_1 and α_2. Additionally, observe that $f_p^*(v_p^1 v_p^2) = 0$. So, $e_{f_p^*}(0) = 2\left(\frac{p-1}{2}\right) + 1 = p$ and $e_{f_p^*}(1) = 2\left(\frac{p-1}{2}\right) = p - 1$. Hence, $\left|e_{f_p^*}(0) - e_{f_p^*}(1)\right| = 1$ and it follows that $B_{p,p}$ admits a Legendre cordial labeling modulo p. □

Example 3.13. Fig. 6 shows the Legendre cordial labeling modulo p of the bistar graph $B_{p,p}$, where $p = 7$.

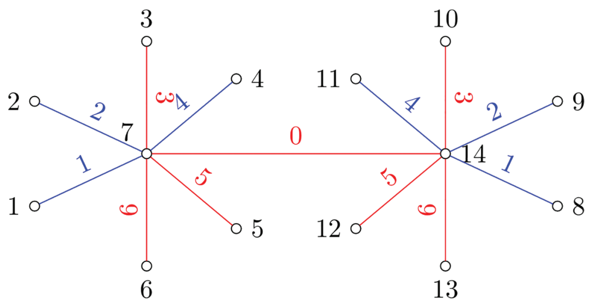

Fig. 6. Legendre cordial labeling modulo 7 of $B_{7,7}$.

Theorem 3.14. *The fan graph F_{p+1} is a Legendre cordial graph modulo p.*

Proof. Let P_p be a path of F_{p+1} with $V(P_p) = \{v_1, v_2, \ldots, v_p\}$. Suppose that v_{p+1} is a new vertex for which $v_i v_{p+1} \in E(F_{p+1})$ for $i = 1, 2, \ldots, p$. Now, let $f\colon V(F_{p+1}) \to \{1, 2, \ldots, p+1\}$ be a bijective function defined by

$$f(v_i) = \begin{cases} i + \dfrac{p-1}{2} & \text{for } i = 1, 2, \ldots, \dfrac{p+1}{2} \\ i - \dfrac{p+1}{2} & \text{for } i = \dfrac{p+3}{2}, \dfrac{p+5}{2}, \ldots, p, \end{cases}$$

and $f(v_{p+1}) = p + 1$. For the edges of P_p,

$$f(v_i) + f(v_{i+1}) \equiv \begin{cases} 2i \,(\bmod p) & \text{for } i = 1, 2, \ldots, \dfrac{p-1}{2} \\ (2i - p)\,(\bmod p) & \text{for } i = \dfrac{p+1}{2}, \dfrac{p+3}{2}, \ldots, p-1. \end{cases}$$

For the remaining the edges of F_{p+1},

$$f(v_i) + f\left(v_{p+1}\right) \equiv \begin{cases} \left(i + \dfrac{p+1}{2}\right)(\bmod p) & \text{for } i = 1, 2, \ldots, \dfrac{p-3}{2} \\ 0\,(\bmod p) & \text{for } i = \dfrac{p-1}{2} \\ \left(i - \dfrac{p-1}{2}\right)(\bmod p) & \text{for } i = \dfrac{p+1}{2}, \dfrac{p+3}{2}, \ldots, p-1. \end{cases}$$

In view of the above labeling, let

$$\alpha = \{\xi : f(v_i) + f(v_{i+1}) \equiv \xi (\bmod p), 0 < \xi < p, i = 1, 2, \ldots, p-1\}, \textit{and}$$

$$\beta = \{\xi : f(v_i) + f(v_{p+1}) \equiv \xi (\bmod p), 0 < \xi < p, i = 1, 2, \ldots, p, i \neq (p-1)/2\}.$$

Thus, $\alpha = \beta = \{1, 2, \ldots, p-1\}$ and by Theorem 2.9, there are $\frac{p-1}{2}$ quadratic residues and $\frac{p-1}{2}$ quadratic nonresidues of p in sets α and β. Additionally, notice that $f_p^*(v_{(p-1)/2} v_{p+1}) = 0$. Hence, $e_{f_p^*}(0) = 2\left(\frac{p-1}{2}\right) + 1 = p$ and $e_{f_p^*}(0) = 2\left(\frac{p-1}{2}\right) = p - 1$. Clearly, $\left|e_{f_p^*}(0) - e_{f_p^*}(1)\right| = 1$. Therefore, F_{p+1} admits a Legendre cordial labeling modulo p. □

Example 3.15. Fig. 7 shows the Legendre cordial labeling modulo p of the fan graph F_{p+1} for the case $p = 11$.

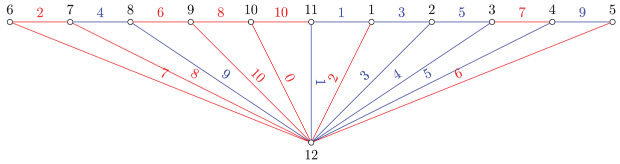

Fig. 7. Legendre cordial labeling modulo 11 of F_{12}.

4. Conclusion

In this study, we proposed a new type of cordial labeling, called Legendre cordial labeling, which incorporates the Legendre symbol into graph labeling. We demonstrated that the path graph P_p, cycle graph C_p, star S_p, tadpole graph $T_{p,p}$, kayak paddle graph $KP_{p,p,p}$, bistar graph $B_{p,p}$, and fan graph F_{p+1} admit a Legendre cordial labeling modulo p, where p is an odd prime. This contributes to the growing field of cordial labeling by linking graph theory with classical number-theoretic ideas. Future work may focus on identifying other graph classes that admit this labeling and investigating possible applications in cryptography, network analysis, and related areas.

Acknowledgment

The authors would like to thank the Department of Science and Technology—Science Education Institute (DOST-SEI) for funding this study.

Conflict of Interest

Authors declared that they do not have any conflict of interest.

References

[1] Chartrand G, Zhang P. *Chromatic Graph Theory*. CRC Press; 2009.
[2] Burton D. *Elementary Number Theory*. 7th ed. McGraw-Hill; 2011.
[3] Rosen K. *Elementary Number Theory and its Application*. 6th ed. Addison Wesley; 2011.
[4] Ghosal G. A study on the development and application of number theory in engineering field. *Int J Inf Sci Comput*. 2020;7(2):109–14. doi: 10.30954/2348-7437.2.2020.5.
[5] Cahit I. Cordial graphs: a weaker version of graceful and harmonious graphs. *Ars Combinatoria*. 1987;23:201–7.
[6] Pedrano A, Rulete R. On the total product cordial labeling on the cartesian product of $P_m \times C_n$, $C_m \times C_n$ and the generalized Petersen graph $P(n, m)$. *Malaya J Matimatik*. 2017;5(3):513–39. doi: 10.26637/mjm503/007.
[7] Suguraman A, Rajesh K. Lucas divisor cordial labeling. *Int J Pure Appl Math*. 2017;113(6):233–41.
[8] Sulayman J, Pedrano A. On fibonacci cordial labeling of some snake graphs. *Eur J Math Stat*. 2023;4(2):29–35. doi: 10.24018/ejmath.2023.4.2.193.
[9] Anto Cathrin Aanisha A, Manoharan R. Mean cordial labeling in graph representations of human anatomy and circulatory systems. *Commun Appl Nonlinear Anal*. 2024;31(7):453–8. doi: 10.52783/cana.v31.1324.
[10] Aravindan SJ, Sasikala D, Divya A. Visualization of cordial graph in human excretion track. *Italian J Pure Appl Math*. 2020;44:460–9.