A Proof for Fermat’s Last Theorem

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ABSTRACT

Fermat’s proposition during 1637 that the Diophantine equation $x^n + y^n = z^n$ where $x, y, z$ and $n$ are integers has no solution for $n > 2$ has come to be known as Fermat’s Last Theorem.

Taking the proofs of the theorem by Fermat and Euler for the index $n = 4$ and $n = 3$, it would suffice to prove the theorem for the index $p$ which is any prime $> 3$.

We consider the equation $r^p + s^p = t^p$ to prove the theorem.

We take another auxiliary equation $x^3 + y^3 = z^3$ to substantiate the proof. Both equations have been combined by means of equivalent equations, into which we have employed the Ramanujan-Nagell equation. Solving the equivalent equations using the Ramanujan-Nagell Equation we prove the theorem.

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1. Preamble

Eminent mathematicians throughout the world have contributed to this theorem for three centuries. At last, in 1994 Professor Andrew Wiles proved the theorem completely using highly advanced number theory [1]–[3].

2. Hypothesis

1) We presume $r, s,$ and $t$ as coprime positive integers in the equation $r^p + s^p = t^p$, where $p$ is any odd prime $> 3$, and find a contradiction.

2) We use auxiliary equation $x^3 + y^3 = z^3$ for supplementing the proof in the first equation. We assign some particular integer values for $x, y$ by which $z^3$ will be an integer and both $z$ and $z^2$ will be irrational.

3) By trials we create equivalent equations to the above two equations.

4) We can get the equivalent values of $2^n, 7$ and $\ell^2$ from the equivalent equations which are substituted in the Ramanujan-Nagell Equations $2^n = 7 + \ell^2$. We take one of the three solutions $2^5 = 7 + 5^2$ or $2^7 = 7 + 11^2$ or $2^{15} = 7 + 181^2$, where $n$ is odd and $\ell$ is larger than 1 for our proof.

Proof. After long trials, we have found the following equations,

$$\left(a\sqrt{5} + b\sqrt{19}\right)^2 + \left(\frac{c\sqrt{2^{6/2} + d\sqrt{F^{1/3}}}}{\sqrt{7}}\right)^2 = \left(e\sqrt{\ell^{7/3}} + f\sqrt{E}\right)^2$$

and

$$\left(a\sqrt{271} - b\sqrt{5}\right)^2 + \left(\frac{c\sqrt{7^{5/3} - d\sqrt{E^{5/3}}}}{\sqrt{2^{3/2}}}\right)^2 = \left(e\sqrt{5^7 - f\sqrt{29}}\right)^2$$

(1)

as the equivalent equations representing $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$ respectively by means of parameters $a, b, c, d, e$ and $f$.

Here $x = 10; y = 19; z^3 = 10^3 + 19^3 = 29 \times 271; 2^n = 7 + \ell^2$ (Ramanujan-Nagell equation).
F any odd prime and \( E = 5 \times 271 \).
From (1), we have

\[
a\sqrt{5} + b\sqrt{19} = \sqrt{x^3}
\]

\[
a\sqrt{271} - b\sqrt{3} = \sqrt{2n^2/p^2}
\]

\[
c\sqrt{2m^2} + d\sqrt{F^{1/3}} = \sqrt{y^3t}
\]

\[
c\sqrt{75/3} - d\sqrt{\epsilon^{5/3}} = \sqrt{F^{5/3}q^p}
\]

\[
e\sqrt{\epsilon^{3/3}} + f\sqrt{E} = \sqrt{z^3}
\]

and \( e\sqrt{5}r - f\sqrt{29} = \sqrt{7^{1/3}p^q} \) \( (7) \)

Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get

\[
a = \left( \sqrt{x^3} + \sqrt{19 \times 2n^2/p^2} \right) / \left( \sqrt{5} + \sqrt{19 \times 271} \right)
\]

\[
b = \left( \sqrt{271}x^3 - \sqrt{2n^2 \times 5^p} \right) / \left( \sqrt{5} + \sqrt{19 \times 271} \right)
\]

\[
c = \left( \sqrt{y^3tF^{5/3}} + \sqrt{F^{3/2}p^q} \right) \left( \sqrt{2n^2 \epsilon^{5/3} + \sqrt{75/3}F^{1/3}} \right)
\]

\[
d = \left( \sqrt{75/3}y^3t - \sqrt{2n^2 \epsilon^{5/3}p^q} \right) \left( \sqrt{2n^2 \epsilon^{5/3} + \sqrt{75/3}F^{1/3}} \right)
\]

\[
e = \left( \sqrt{29z^3} + \sqrt{7^{1/3}E^{1/p}} \right) \left( \sqrt{29 \times \epsilon^{7/3} + \sqrt{5}Er} \right)
\]

and \( f = \left( \sqrt{5r^3} - \sqrt{7^{1/3}E^{7/3}p^q} \right) \left( \sqrt{29 \times \epsilon^{7/3} + \sqrt{5}Er} \right) \)

From (3) & (4), we have

\[
\sqrt{2n^2} \times \sqrt{2m^2} = \left( a\sqrt{271} - b\sqrt{3} \right) \left( \sqrt{y^3t} + d\sqrt{F^{3/2}} \right) / \left( c\sqrt{p^q} \right)
\]

i.e., \( 2^n = \left( (a) \sqrt{271}y^3t + (ad) \sqrt{271 \times F^{1/3}} - (b) \sqrt{y^3st} + (bd) \sqrt{F^{1/3}x} \right) / \left( c\sqrt{p^q} \right) \)

From (5) & (7), we have

\[
\sqrt{7^{1/3}} \times \sqrt{75/3} = \left( \sqrt{F^{5/3}p^q} + d\sqrt{\epsilon^{5/3}} \right) \left( e\sqrt{5r} - f\sqrt{29} \right) / \left( c\sqrt{p^q} \right)
\]

i.e., \( 7 = \left( (e) \sqrt{F^{5/3} \times 5r} - (f) \sqrt{29 \times F^{5/3}p^q} + (de) \sqrt{5r\epsilon^{5/3}} - (df) \sqrt{29\epsilon^{5/3}} \right) / \left( c\sqrt{p^q} \right) \)

From (5) & (6), we have

\[
\sqrt{\epsilon^{3/3}} \times \sqrt{\epsilon^{5/3}} = \left( c\sqrt{7^{1/3}} - \sqrt{F^{5/3}p^q} \right) \left( \sqrt{z^3} - f\sqrt{E} \right) / \left( de \right)
\]

i.e., \( \epsilon^2 = \left( (c) \sqrt{7^{5/3}z^3} - (cf) \sqrt{7^{5/3}E} - \sqrt{F^{5/3}p^qz^3} + (f) \sqrt{F^{5/3}E}p^q \right) / \left( de \right) \)

By obtaining the equivalent values of \( 2^n \), \( 7 \) and \( \epsilon^2 \) in the equation \( 2^n = 7 + \epsilon^2 \) after multiplying throughout by \( \left( (cde) \sqrt{\epsilon^{9/3}} \right) \), we get

\[
\left[ (de) \sqrt{p^q} \right] \left( (a) \sqrt{271}y^3t - (ad) \sqrt{271 \times F^{5/3}} - (b) \sqrt{y^3st} + (bd) \sqrt{F^{1/3}x} \right) \]

\[= \left( (de) \sqrt{p} \right) \left( (e) \sqrt{5 \times F^{5/3}p^q} - (f) \sqrt{29 \times F^{5/3}p^q} + (de) \sqrt{5r\epsilon^{5/3}} - (df) \sqrt{29\epsilon^{5/3}} \right) \]

\[+ \left( (c) \sqrt{p^q} \right) \left( (c) \sqrt{7^{5/3}z^3} - (cf) \sqrt{7^{5/3}E} - \sqrt{F^{5/3}p^qz^3} + (f) \sqrt{F^{5/3}E}p^q \right) \]
We are working on the rational terms in (8). The multiplying throughout by
\[
\left( \sqrt{5s} + \sqrt[3]{19 \times 271} \right) \left( \sqrt{2^{m/2}x^{3/3} + \sqrt[3]{7^{5/3}F^{1/3}}} \right) \left( \sqrt{29 \times \ell^{1/3} + \sqrt[6]{Ev}} \right)^2
\]
to get ridom denominators and also \(\sqrt{s}t\) so that a few rational terms in (8) are obtained.

I term in LHS of (8), on due multiplications and giving the values for \(ade\)
\[
= \sqrt{p^t} \sqrt[6]{271y^3 \left( \sqrt[3]{2^{m/2}x^{3/3} + \sqrt[3]{7^{5/3}F^{1/3}}} \right) \left( \sqrt{29 \times \ell^{1/3} + \sqrt[6]{Ev}} \right) \sqrt{s}t}
\]
on multiplying by
\[
\left( \sqrt{5s} + \sqrt[3]{19 \times 2^{n/2}p^r} \right) \left( \sqrt{7^{5/3}y^3} - \sqrt[3]{2^{m/2}F^{3/3}g^p} \right) \left( \sqrt{29z^3} - \sqrt[3]{7^{1/3}Ev} \right)
\]
we get
\[
\left( (p^t+1) \sqrt{Fy} \right) \sqrt{271y} \sqrt{3} \times 19 \times 2^{n/2}p^r \sqrt{3} \times 19 \times 2^{n/2}p^r \left( \sqrt{29z^3} - \sqrt[3]{7^{1/3}Ev} \right)
\]
which is rational since \(y = 19; E = 5 \times 271\).

II term in LHS of (8), on due multiplications and giving the values for \(ad^2e\)
\[
= \left( \sqrt{271} \times F^{1/3}p^r \right) \left( \sqrt{29 \ell^{1/3} + \sqrt[3]{5Ev}} \right) \left( \sqrt{x^3s} + \sqrt[3]{19 \times 2^{n/2}p^r} \right) \sqrt{s}t
\]
on multiplying by
\[
\left( \sqrt{7^{5/3}y^3} + F^{5/3}p^r \sqrt{2^{n/2}F^{3/3}g^p} \right) \left( \sqrt{29z^3} - \sqrt[3]{7^{1/3}Ev} \right)
\]
we get
\[
\left( (p^t+1) \sqrt{Fy} \right) \sqrt{271y} \sqrt{3} \times 19 \times 2^{n/2}p^r \sqrt{3} \times 19 \times 2^{n/2}p^r \left( \sqrt{29z^3} - \sqrt[3]{7^{1/3}Ev} \right)
\]
which is rational since \(y = 19; E = 5 \times 271\).

III term in LHS of (8), on due multiplications and giving the values for \(bde\)
\[
= \left( \sqrt{p^t+1} \sqrt{x^3s} \sqrt{29 \ell^{1/3} + \sqrt[3]{5Ev}} \right) \left( \sqrt{2^{m/2}x^{3/3} + \sqrt[3]{7^{5/3}F^{1/3}}} \right) \sqrt{s}t
\]
on multiplying by
\[
\left( \sqrt{271} \times x^3 - \sqrt[3]{2^{n/2} \times 5p^r} \right) \left( \sqrt{7^{5/3}y^3} - \sqrt[3]{2^{m/2}F^{3/3}g^p} \right) \left( \sqrt{29z^3} + \sqrt[3]{7^{1/3}Ev} \right)
\]
we get
\[
\left( (p^t+1) \sqrt{Fy} \right) \sqrt{271} \times 2^{n/2} \times 5p^r \sqrt{3} \times 19 \times 2^{n/2} \times 5p^r \left( \sqrt{29z^3} + \sqrt[3]{7^{1/3}Ev} \right)
\]
which is rational since \(y = 19; E = 5 \times 271\).

Since \(\sqrt{E} = \sqrt{5 \times 271}\)
\[
\sqrt{5 \times 271p^t} = 5 \sqrt{29 \times 271p^t}, \text{ which will be irrational if } r \text{ and } t \text{ are coprimes to } 29 \text{ and } 271; \text{ otherwise we have the choice of assigning alternative values, for example, to } x = 2 \times 3; y = 13; \text{ and } z^3 = 6^3 + 13^3 = 19 \times 127 \text{ such that } r \text{ and } t \text{ are coprime to odd prime factors in } x, y \text{ and } z^3.
\]
IV term in LHS of (8), on due multiplications and giving the values for \( \{bd^2c\} \)

\[
= \sqrt{F^{1/3}sp^3} \left( \sqrt{29}r^{3/2} + \sqrt{5}Er \right) \left( \sqrt{271}r^3 + \sqrt{5} \times 2^{m/2}r^3 \right) \sqrt{st}
\]

\[
\times \left( 7^{5/3}y^3 I + F^{5/3}y^3 \sqrt{23n} - 2\sqrt{7^{5/3}y^3 I \times 2^{m/2}F^{5/3}y^3} \right) \left( \sqrt{29}z^3 - \sqrt{7^{1/3}Ep^3} \right)
\]

on multiplying by

\[
\left( \sqrt{F^{1/3}st^6} \times 5Er \left( -\sqrt{5} \times 2^{m/2}r^3 \right) \sqrt{st} \left( -2\sqrt{7^{5/3}y^3 I \times 2^{m/2}F^{5/3}y^3} \right) \sqrt{7^{1/3}Ep^3} \right)
\]

we get

\[
\left( 2^{n+1} \times 7 \times 5FE \sqrt{y^3} \sqrt{(rs)^{p+1}} \left( p^{p+1} \right) \right)
\]

which will be irrational if \( s \) is coprime to \( y \), where \( y = 19 \); otherwise we have the choice in having alternative suitable values for \( x \), \( y \) and \( z^3 \) such that \( s \) is coprime to \( y \).

I term in RHS of (8), on due multiplications and giving the values for \( \{dce^2\} \)

\[
= \left( \sqrt{5e^{3/2}sp^3} \right) \left( \sqrt{5}r + \sqrt{19 \times 271} \right) \left( \sqrt{2^{m/2}e^{5/2}} + \sqrt{7^{5/3}F^{1/3}} \right) \sqrt{st}
\]

\[
\times \left( \sqrt{7^{5/3}y^3 I - 2^{m/2}E^{5/3}y^3s} \right) \left( (29z^3) + (7^{1/3}Ep^3) + 2\sqrt{29z^3} \sqrt{7^{1/3}Ep^3} \right)
\]

on multiplying by

\[
\left( \sqrt{5e^{3/2}sp^3} \sqrt{19 \times 271} \sqrt{7^{5/3}F^{1/3} \sqrt{st} \sqrt{7^{5/3}y^3 I}} \left( 7^{1/3}Ep^3 \right) \right)
\]

we get the irrational term

\[
\left( 7^2FE^{p+1} \sqrt{(rs)^{p+1}} \sqrt{19r^3} \sqrt{5} \times 271 \right)
\]

II terms in RHS of (8), on due multiplications and giving the values for \( \{def\} \)

\[
= \left( -\sqrt{29}F^{5/3}sp^3 \sqrt{5r} + \sqrt{19 \times 271} \right) \left( \sqrt{2^{m/2}e^{5/2}} + \sqrt{7^{5/3}F^{1/3}} \right) \sqrt{st}
\]

\[
\times \left( \sqrt{7^{5/3}y^3 I - 2^{m/2}E^{5/3}y^3s} \right) \left( \sqrt{29z^3} + \sqrt{7^{1/3}Ep^3} \right) \left( \sqrt{5z^3r} - \sqrt{7^{1/3}E^{5/3}p^3} \right)
\]

on multiplying by

\[
\left( -\sqrt{29}F^{5/3}sp^3 \sqrt{5r} \sqrt{19 \times 271} \sqrt{7^{5/3}F^{1/3} \sqrt{st} \sqrt{7^{5/3}y^3 I}} \sqrt{7^{1/3}Ep^3} \left( -\sqrt{7^{1/3}E^{5/3}p^3} \right) \right)
\]

we get

\[
\left( 7^2FE^{p+1} \sqrt{sp^3} \sqrt{19 \times 29 \times 271Ey^3p^6} \right)
\]

which is irrational \( \ell \) being \( > 1 \).

III terms in RHS of (8), on due multiplications and giving the values for \( \{d^2c^2\} \)

\[
= \sqrt{r^{p+1} \sqrt{5e^{5/2}}} \left( \sqrt{5r} + \sqrt{19 \times 271} \right) \left( (7^{5/3}y^3 I) + (F^{5/3}y^3 \sqrt{23n}) - 2\sqrt{7^{5/3}y^3 I \times 2^{m/2}F^{5/3}y^3} \right) \sqrt{st}
\]

\[
\times \left( 29z^3 + 7^{1/3}Ep^3 + 2\sqrt{29z^3} \sqrt{7^{1/3}Ep^3} \right)
\]

There is no rational part in this term.
IV term in RHS of (8), on due multiplications and giving the values for \( \{d^2(ef')\} \)
\[
\left(-\sqrt{29\ell^{5/3}p^r}\right) \left(\sqrt{5s} + \sqrt{19 \times 271}\right) \left(\ell^{5/3}y^3t + \left(F^{5/3}g^3\sqrt{2s^3} - 2\sqrt{2^3/2F^{5/3}g^3\sqrt{5^{5/3}}y^3t}\right)\sqrt{5t} \times \left(\sqrt{29\ell^{5/3}p^r} + \sqrt{7^{5/3}}Er^p\right) \left(\sqrt{5s} - \sqrt{7^{5/3}\ell^{5/3}}p^r\right) \right.
\]
on multiplying by
\[
\left(-\sqrt{29\ell^{5/3}p^r}\right) \sqrt{5s} \left(\ell^{5/3}y^3t\right) \sqrt{5t} \sqrt{7^{5/3}Er^p} \left(-\sqrt{7^{5/3}\ell^{5/3}}p^r\right) \right]
\]
we get
\[
\left(\ell^{5/3}y^3t\sqrt{5 \times 29} \times Er^p\right)\]
which does not give any irrational term.

V term in RHS of (8), on due multiplications and giving the values for \( \{e^2\} \)
\[
\sqrt{7^{5/3}z^3p^r} \left(\sqrt{5s} + \sqrt{19 \times 271}\right) \left(\ell^{5/3}y^3t + \left(F^{2}s^3\right) + 2F\sqrt{\ell^{5/3}y^3g^3t}\right) \sqrt{5t} \times \left(29\ell^{5/3} + (5Er) + 2\sqrt{29\ell^{5/3}\sqrt{5Er}}\right)\]
which does not give any rational term.

VI terms in RHS of (8), on due multiplications and giving the values for \( \{e^3f\} \)
\[
\left(-\sqrt{7^{5/3}Er^p}\right) \left(\sqrt{5s} + \sqrt{19 \times 271}\right) \left(\sqrt{2^3/2F^{5/3} + \sqrt{7^{5/3}F^{1/3}}}\right) \sqrt{5t} \times \left(\ell^{5/3}y^3t + \left(F^2s^3\right) + 2\sqrt{\ell^{5/3}y^3g^3t}\right) \sqrt{5s} \sqrt{29\ell^{5/3} + \sqrt{7^{5/3}\ell^{5/3}}p^r}\]
on multiplying by
\[
\left(-\sqrt{7^{5/3}Er^p}\right) \sqrt{5s} \sqrt{29\ell^{5/3} + \sqrt{7^{5/3}\ell^{5/3}}p^r} \left(\ell^{5/3}y^3t\right) \left(-\sqrt{7^{5/3}\ell^{5/3}}p^r\right) \right]
we get
\[
\left(\ell^{4/3}y^3\sqrt{5 \times 29} \times Er^p\right)\]
which is irrational as we discussed earlier.

VII terms in RHS of (8), on due multiplications and giving the values for \( \{e^3\} \)
\[
\left(-\sqrt{F^{5/3}z^3p^r}\right) \left(\sqrt{5s} + \sqrt{19 \times 271}\right) \left(\ell^{5/3}y^3t + \left(F^2s^3\right) + 2\sqrt{2^3/2F^{5/3} + \sqrt{7^{5/3}F^{1/3}}}\right) \sqrt{5t} \times \left(29\ell^{5/3} + (5Er) + 2\sqrt{29\ell^{5/3}\sqrt{5Er}}\right)\]
which is devoid of rational parts.

VIII terms in RHS of (8), on due multiplications and giving the values for \( \{ef\} \)
\[
\left(\sqrt{F^{5/3}Er^p}\right) \left(\sqrt{5s} + \sqrt{19 \times 271}\right) \left(\ell^{5/3}y^3t + \left(F^2s^3\right) + 2\sqrt{2^3/2F^{5/3} + \sqrt{7^{5/3}F^{1/3}}}\right) \sqrt{5t} \times \left(29\ell^{5/3} + (5Er) + 2\sqrt{29\ell^{5/3}\sqrt{5Er}}\right)\]
on multiplying by
\[
\left(\sqrt{F^{5/3}Er^p}\right) \sqrt{5s} \sqrt{29\ell^{5/3} + \sqrt{7^{5/3}\ell^{5/3}}p^r} \sqrt{5s} \sqrt{5t} \sqrt{5Er} \left(-\sqrt{7^{5/3}\ell^{5/3}}p^r\right) \right]
we get
\[
\left(-7\ell^2\sqrt{5E}\right) \sqrt{(r^p)^{p+1}} \left(p^r + \sqrt{y^3}\right)\]

which will be irrational if \( s \) coprime to \( y = 19 \).

Totaling the rational terms on LHS of (8)

\[
= \left(2^n \times 7Fy \sqrt{19y}\sqrt{5 \times 271E \sqrt{(rs)^{p+1} (p^{p+1})}}\right) \text{ (combining I and II terms)}
\]

there is no rational term on the RHS of (8).

Equating both sides, the rational terms in (8) after dividing both sides by

\[
\left(2^n \times 7Fy \sqrt{19y}\sqrt{5 \times 271E} \right)
\]

we get

\[
\left(\sqrt{(rs)^{p+1}}\right) = 0
\]

\[\therefore \text{ rst } = 0.\]

This disproves the hypothesis that all \( r, s, t \) are positive integers in \( r^p + s^p = t^p \).

3. Conclusion

Since (8) was obtained from the equivalent equation to Fermat’s Equations, the result we get from (8) proves the theorem.

Conflict of Interest

Author declares no conflict of interest.

References

