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RESEARCH ARTICLE

Comparative Analysis of GARCH-Based Volatility Models of Financial Market Volatility: A Case of Nairobi Security Market PLC, Kenya

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ABSTRACT

This paper conducted a comprehensive comparative analysis of various GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models to forecast financial market volatility, with a specific focus on the Nairobi Stock Exchange Market. The examined models include symmetric and asymmetric GARCH types, such as sGARCH, GJR-GARCH, AR (1) GJG-GARCH, among others. The primary objective is to identify the most suitable model for capturing the complex dynamics of financial market volatility. The study employs rigorous evaluation criteria, including the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Mean Error (ME), and Root Mean Absolute Error (RMAE), to assess the performance of each model. These criteria facilitate the selection of the optimal model for volatility forecasting. The analysis reveals that the GJR-GARCH (1,1) model emerges as the best-fit model, with AIC and BIC values of −5.5008 and −5.4902, respectively. This selection aligns with the consensus in the literature, highlighting the superiority of asymmetric GARCH models in capturing volatility dynamics. The comparison also involves symmetric GARCH models, such as sGARCH (1,1), and other asymmetric models like AR (1) GJG-GARCH. While these models were considered, the GJR-GARCH (1,1) model demonstrated superior forecasting capabilities. The study emphasizes the importance of accurate model selection and the incorporation of asymmetry in volatility modeling. The research provides essential insights into financial market volatility modeling and forecasting using both asymmetric and symmetric GARCH models. These findings have significant implications for government policymakers, financial institutions, and investors, offering improved tools for risk assessment and decision-making during periods of market turbulence.

Keywords: Financial market volatility, GARCH models, model selection, Nairobi Stock Exchange Market.

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1. Introduction

Financial market volatility is a fundamental driver of investment strategies, risk management practices, and financial decision-making processes [1]. It is a dynamic force that shapes the behavior of financial markets, influencing asset prices, trading volumes, and investor sentiment [2]. As investors and market participants continually strive to navigate the complex and ever-changing landscape of financial markets, understanding and predicting volatility has become a critical aspect of their success. This paper aims to provide an in-depth exploration and comparative analysis of GARCH (Generalized Autoregressive Conditional Heteroskedasticity) based volatility models, which have emerged as powerful tools for forecasting and managing financial market volatility. Volatility, in the context of financial markets, refers to the degree of variation and fluctuations in the prices of financial assets over time, which measure of the uncertainty and risk inherent in financial markets, reflecting the extent to which asset prices deviate from their average or expected values [3]. High levels of volatility often indicate market turbulence, while low volatility can signify periods of stability and predictability [4].

The importance of understanding and forecasting financial market volatility cannot be overstated. For investors, the ability to anticipate and react to changes in volatility is critical for optimizing investment portfolios and managing risk [5]. Volatility forecasts assist in asset allocation decisions, helping investors determine the appropriate mix of assets to achieve their financial goals while minimizing exposure to unnecessary risk [6]. Risk management practitioners rely heavily on volatility models to develop strategies and tools for hedging against adverse market movements. These models enable them to estimate potential losses and design risk mitigation strategies that protect portfolios from unexpected shocks. Furthermore, financial institutions, policymakers, and central banks closely monitor market volatility as part of their broader efforts to ensure financial stability and economic growth. In other words, time series modeling approaches have grown in popularity in modeling time series due to increased demand for forecasts in economic and financial time series data [7].

Sudden spikes in volatility can have far-reaching consequences, triggering market crises and systemic risks that can affect economies on a global scale [8]. GARCH models have, therefore, emerged as a cornerstone in the field of volatility modeling. They are particularly well-suited to capture the time-varying nature of financial market volatility. GARCH models extend the traditional notion of volatility as a constant and introduce the concept of conditional heteroskedasticity, which means that volatility can change over time based on past observations [9]. This dynamic approach allows GARCH models to provide more accurate and nuanced volatility forecasts compared to simpler models. Thus, this paper will delve into the intricacies of GARCH-based volatility models, exploring their underlying principles, empirical applications, and comparative performance. By shedding light on the strengths and weaknesses of these models, it aims to contribute to the ongoing dialogue surrounding volatility forecasting and enhance our understanding of how investors and institutions can better navigate the complexities of financial markets in an increasingly uncertain world.

According to Wang and Lu [10], financial market volatility refers to the degree of variation in the prices of financial assets over time. It reflects the uncertainty and risk associated with financial markets, estimated using GARCH models. GARCH models are statistical models used to estimate and forecast the volatility of financial returns. These models take into account the conditional heteroskedasticity, or time-varying volatility, of financial time series data.

Floros investigated the application of GARCH-type models to effectively model volatility of financial market risk. The study harnesses daily data sourced from the Egyptian market (CMA General Index) and the Israeli market (TASE-100 Index) and employed a diverse range of time series methodologies, encompassing the basic GARCH model, as well as more advanced variations such as exponential GARCH, threshold GARCH, asymmetric component GARCH, component GARCH, and the power GARCH model. The study established quite compelling evidence that these models aptly characterize the daily return patterns in both the Egyptian and Israeli markets [11]. One of the central findings from the investigation pertains to the relationship between market risk and returns. Contrary to conventional assumptions, the research concludes that heightened market risk does not necessarily translate into increased returns. This intriguing insight is consistent across both markets under scrutiny. Notably, the CMA Index in Egypt emerges as the most volatile series, primarily attributed to the considerable uncertainty that persisted in both price movements and economic conditions throughout the observed period. These findings hold substantial implications for financial practitioners, particularly for financial managers and modelers engaged in the analysis of international markets.

In a different study, Ugurlu *et al.* [12] investigated the application of GARCH-type models in assessing the volatility of stock market returns across five European emerging countries: Bulgaria (SOFIX), Czech Republic (PX), Poland (WIG), Hungary (BUX), and Turkey (XU100). Utilizing daily data, their study explored the dynamics in these emerging financial markets. Findings revealed that GARCH, GJR-GARCH, and EGARCH effects are notably present in the returns of PX, BUX, WIG, and XU100, underscoring the significance of conditional volatility modeling. However, it is noteworthy that such effects are not statistically significant for SOFIX, suggesting a unique characteristic in its volatility behavior.

Orhan and Köksal [13], on the other hand conducted a comprehensive comparison of various GARCH models to assess the quantification of Value at Risk (VaR) during times of financial stress times. The research collected stock market index data from both emerging markets (Brazil and Turkey) and developed markets (Germany and the USA), focusing on the period encompassing the global financial crisis. A multitude of GARCH model specifications were employed to compute VaR values, and these assessments were subsequently compared against realized returns using Kupiec and Christoffersen Tests. Additionally, the study calculated Quadratic Losses to evaluate the performance of the GARCH specifications in VaR estimation. The findings indicate that the ARCH specification emerges as the top performer, followed closely by GARCH (1,1), and that the student's t distribution slightly outperforms the Normal distribution. Conversely, the poorest performers are found to be the Non-Linear Power GARCH and Non-Linear Power GARCH with a shift model. All GARCH estimations were conducted using STATA, utilizing the Maximum Likelihood estimation method.

In a study conducted by Katsiampa [14], the focus was on estimating Bitcoin price volatility by employing various GARCH model variants. The results revealed that the AR (1)-CGARCH (1,1) model maximized the log-likelihood value, and notably, all three information criteria also favored this model. Furthermore, the parameter estimates within the AR (1)-CGARCH (1,1) model were found to be statistically significant. Diagnostic tests, specifically the ARCH (5) and ARCH (10) Q tests applied to both squared residuals and squared standardized residuals of the AR (1)-CGARCH (1,1) model, indicated that this model was well-suited for describing Bitcoin price returns. The tests did not reject the hypotheses of no remaining ARCH effects and no autocorrelation. Although the residuals of this model deviated from normality, the Jarque-Bera test showed a considerable improvement compared to the corresponding values for the returns. In summary, the AR-CGARCH model was identified as an appropriate tool for characterizing Bitcoin price return volatility. This finding aligns with a study by Bouoiyour and Selmi [15], which also suggested that the CMT-GARCH model, featuring both transitory and permanent components, was optimal for modeling Bitcoin price dynamics during the period from December 2010 to December 2014. Lim and Sek [16] conducted an empirical analysis to model the volatility of the stock market in Malaysia using GARCH-type models, including both symmetric and asymmetric GARCH models. The performance of these models was compared based on three statistical error measures: mean squared error, root mean squared error, and mean absolute percentage error, for both in-sample and out-of-sample analyses. Additionally, factors contributing to stock market movements were identified. The data covered the period from January 1990 to December 2010, divided into three distinct time frames: pre-crisis 1997, the crisis period, and post-crisis 1997. The results indicated that the performance of symmetric and asymmetric GARCH models varied across different time frames. Generally, during normal periods (pre- and post-crisis), symmetric GARCH models outperformed asymmetric GARCH models. However, during periods of market fluctuation (the crisis period), asymmetric GARCH models were preferred. Furthermore, the findings revealed that exchange rates and crude oil prices had significant impacts on Malaysia's stock market volatility during the pre-crisis and post-crisis periods, but their impact was not significant during the crisis period.

In many cases, comparisons of various GARCH models are based on the distribution under consideration. Gao *et al.* [17] employed the MCMC (Markov Chain Monte Carlo) method to conduct a comparative analysis of three distinct GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models: N-GARCH, T-GARCH, and GED-GARCH, both in univariate and multiple cases. In the univariate case, Gao *et al.* [17] scrutinized the kurtosis coefficients and standard deviations by contrasting the model-derived autocorrelation with the actual autocorrelation. Our findings established a hierarchy of model performance, with GED-GARCH outperforming T-GARCH, and T-GARCH surpassing N-GARCH. Extending their analysis to the multiple case scenario, Heston and Nandi [18] employed metrics such as Adaptive Mean Absolute Deviation and Adaptive Root of Mean Square Error for comparison. Remarkably, the results mirrored those from the univariate case, with GED-GARCH emerging as the superior model, followed by T-GARCH, and N-GARCH trailing behind. This consistent outcome underscores the effectiveness of GED-GARCH, particularly when compared to other GARCH models based on different distributions. From the results, it becomes evident that GED-GARCH stands out as the preferred GARCH model, prompting consideration for future adjustments in the expanded form of N-GARCH.

Recent empirical research has demonstrated the effectiveness of GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models in describing option prices. Pricing options necessitates understanding the risk-neutral cumulative return distribution, typically lacking known analytical forms. Consequently, computationally intensive numerical methods are essential for pricing calculations. According to Hsieh and Ritchken [19] who introduced a specific GARCH structure that enables analytical solutions for pricing European options, substantiated by empirical evidence. However, their model belongs to the affine family, which contrasts with the prevalent non-affine GARCH models explored in most studies. Hsieh and Ritchken [19] scrutinized Heston and Nandi's model to assess whether the focus on affine family models entails any drawbacks. Their investigation corroborates Heston and Nandi's findings, confirming that their model effectively accounts for a substantial portion of the volatility smile. Nevertheless, Hsieh and Ritchken [19] demonstrated that a straightforward non-affine NGARCH (Nonlinear GARCH) option model outperforms Heston and Nandi's model in mitigating biases in pricing residuals across all moneyness and maturity categories, particularly for out-of-the-money contracts.

Many economic and financial applications, such as portfolio optimization and risk management, now require modelling and forecasting the volatility of a financial time series [20]. Volatility and leptokurtosis can be captured using symmetric-GARCH models. The models, however, do not account for leverage effects, volatility clustering, or the thick tail property of high-frequency financial time series. In their study, Ndege *et al.* [20] applied asymmetric-GARCH type models to Kenyan exchange rate in order to address the drawbacks of symmetric-GARCH type models. The models of the asymmetric Conditional Heteroskedasticity class were compared in the study: EGARCH, TGARCH, APARCH, GJR-GARCH, and IGARCH. The Central Bank of Kenya website provided secondary statistics on the exchange rate from January 1993 until June 2021. The best fit model is chosen based on the parsimony of the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), the Log-Likelihood criterion, and the minimization of prediction production errors (Mean error [ME] and Root Mean Absolute error [RMAE]). For the exchange rate data, the best variance equation was the APARCH (1,1)-ARMA (3,0) model with a skewed normal distribution (AIC = −4.6871, BIC = −4.5860). Volatility clustering was seen in exchange rate data, indicating the presence of the leverage effect. Kenya's estimated exchange rate volatility narrows over time, showing long-term exchange rate stability. Overall, the number of studies reviewed share some commonality on the GARCH model preference, thus far show a preference for asymmetric-GARCH models over symmetric-GARCH models. However, the current study does not agree much with the prevalent view and uses asymmetric-GARCH models to analyze financial time series data.

The use of asymmetric-GARCH models is supported by their demonstrated advantage in capturing the subtle dynamics of financial time series, particularly stock prices. It is worth acknowledging that these models are well-suited to account for the asymmetry in volatility's reaction to positive and negative shocks, which is an important feature in understanding stock prices volatility fluctuations and the risks associated with them. This study will, however, apply various variants symmetric and asymmetric GARCH models including; sGARCH with constant mean, GARCH with sstd, GJR-GARCH, AR (1) GJG-GARCH, GJG-GARCH in mean. The following specific objectives guided the study:

1. To fit asymmetric symmetric GARCH type models (sGARCH with constant mean, GARCH with sstd, GJR-GARCH, AR (1) GJG-GARCH, GJG-GARCH in mean.) to the Nairobi Securities Exchange PLC, stock prices.
2. To identify the best symmetric and asymmetric-GARCH type model that best fits the Nairobi Securities Exchange PLC, stock prices.
3. To forecast the Nairobi Securities Exchange PLC, stock prices using the best asymmetric-GARCH type model.

2. Methodology

2.1. Research Design

The research design employed in this study is of a descriptive nature. Within this descriptive research design, the primary objective is to investigate and elucidate the inherent characteristics of financial time series data. These characteristics encompass notable phenomena such as volatility clustering, negative kurtosis, and excess skewness. To gain insights into these attributes, the study harnessed a multifaceted approach that encompassed data visualization techniques, alongside the application of descriptive and inferential statistics. This comprehensive methodology allowed for a detailed exploration and quantification of the stylized properties exhibited by the financial time series data under scrutiny.

2.2. Data Collection

The data collection process for this study involved sourcing daily stock price information for the Nairobi Stock Market PLC from the official Nairobi Security Exchange Market [https://www.nse.co.ke/]. The dataset spans a substantial period, commencing from January 1, 2007, and extending up to September 23, 2023. This extensive timeframe encompasses a diverse array of market conditions and economic events, making it well-suited for comprehensive analysis. The choice of utilizing data from the Nairobi Stock Exchange Market ensures the reliability and authenticity of the stock price information, as it originates from an established and authoritative source within the financial domain. This data constitutes a valuable resource for assessing the volatility and dynamics of the Nairobi Stock Market over the examined period. The decision to employ daily stock prices is particularly advantageous, as it enables the capture of intraday fluctuations and market movements, providing a granular view of market behavior. Additionally, the lengthy time frame considered in this dataset facilitates the examination of long-term trends and the evaluation of the performance of various volatility models under diverse market conditions.

2.3. Data Analysis

The R statistical software served as the primary analytical tool for this study's data analysis phase. The initial analytical steps encompassed a range of procedures, including descriptive statistics, trend analysis, and stationarity testing. These preliminary analyses provided crucial insights into the characteristics and behavior of the dataset under examination. Subsequently, the study focused on fitting the selected symmetric and asymmetric-GARCH models, sGARCH with constant mean, (GARCH with sstd, GJR-GARCH, AR (1) GJG-GARCH), and GJG-GARCH in mean, to the stationary returns of closing stock prices. The RUGARCH package within R was employed to facilitate the estimation and implementation of these models [20]. This comprehensive approach allowed for a rigorous evaluation of the models' performance in capturing the complex volatility dynamics present within the financial time series data.

2.3.1. Models Fitting

Within the framework of the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, the conditional variance of the return series is expressed as a function involving several components [21]. These components encompass a constant term, historical volatility observations, and the prior forecasted variance. To ascertain the model's parameters, the study employed the Maximum Likelihood Estimation (MLE) approach, a statistical method recognized for its effectiveness in estimating model parameters. This approach enabled the precise determination of the GARCH model's coefficients, which are pivotal in capturing the dynamic nature of volatility in financial time series data. By leveraging MLE, the study sought to optimize the model's ability to characterize and forecast conditional variances, thus enhancing the comprehension and predictive power of the GARCH model in the context of financial market volatility analysis.

2.3.1.1. sGARCH with Constant Mean

Consider a univariate time series y_t and t represents discrete time intervals. The GARCH with sstd model can be represented as:

$$y_t = \mu + \varepsilon_t, \tag{1}$$

where
y_t: The observed value of the time series at time t.
μ: The conditional mean of y_t.
ε_t: The innovation or error term at time t, which follows a student's t-distribution with a specific degree of freedom, denoted as ν.

The conditional variance of ε_t in the GARCH-sstd model is modeled as follows:

$$\sigma^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{2}$$

where
σ^2: The conditional variance of ε_t at time t.
ω: Constant term representing the long-term or unconditional variance.
α: The coefficient associated with the lagged squared error term ε_{t-1}^2 which captures the short-term impact of past shocks on volatility.
β: The coefficient associated with the lagged conditional variance σ_{t-1}^2, which captures the long-term persistence in volatility.
ε_t: It follows a normal distribution with mean zero and constant variance.

2.3.1.2. GARCH with Student's t-Distribution

The conditional variance of ε_t in the GARCH-sstd model is modeled as follows, where the error term follows a student's t-distribution:

$$\sigma^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{3}$$

where
σ^2: The conditional variance of ε_t at time t.
ω: Constant term representing the long-term or unconditional variance.
α: The coefficient associated with the lagged squared error term ε_{t-1}^2 which captures the short-term impact of past shocks on volatility.

β: The coefficient associated with the lagged conditional variance σ_{t-1}^2, which captures the long-term persistence in volatility.
ε_t: It follows a student's t-distribution with a specific degree of freedom.

2.3.1.3. GJR-GARCH

GJR-GARCH, which stands for Generalized Autoregressive Conditional Heteroskedasticity with a GARCH-in-mean term, is an extension of the standard GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model commonly used in financial econometrics and time series analysis. In a GJR-GARCH model, the primary objective is to capture both the conditional volatility (heteroskedasticity) of a time series and the potential asymmetry in the response of volatility to past shocks or innovations.

The GJR-GARCH model is represented by the following equation:

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \ldots + \beta_1\sigma_{t-1}^2 + \beta_2\sigma_{t-2}^2 + \ldots + \gamma\varepsilon_{t-1}^2 I_{\varepsilon_{t-1<0}}, \quad (4)$$

where
σ_t^2: The conditional variance of the time series at time t − 1.
ω: A constant term.
α, β, γ: Parameters to be estimated.
ε_{t-1}: The innovation or error term at time t − 1.
$I_{\varepsilon_{t-1<0}}$: An indicator function that takes the value 1 if $\varepsilon_{t-1} < 0$, is less than zero (indicating a negative shock), and 0 otherwise.

2.3.1.4. AR (1) GJG-GARCH

Consider a univariate time series y_t where t represents discrete time intervals. The AR (1) GJR-GARCH model combines autoregressive (AR) dynamics with a GJR-GARCH component to capture conditional volatility with asymmetry.

i) Conditional Mean Equation AR (1):

$$y_t = \mu + \varnothing(y_{t-1} - \mu) + \varepsilon_t \quad (5)$$

where
y_t: The observed value of the time series at time t.
μ: The conditional mean of y_t.
$\varnothing$: The autoregressive coefficient, representing the impact of the lagged value y_{t-1} on the current value.
ε_t: The error term.

ii) Conditional Variance Equation (GJR-GARCH)

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \ldots + \beta_1\sigma_{t-1}^2 + \beta_2\sigma_{t-2}^2 + \ldots + \gamma\varepsilon_{t-1}^2 I_{\varepsilon_{t-1<0}}, \quad (6)$$

where
σ_t^2: The conditional variance of the time series at time t − 1.
ω: A constant term.
α, β, γ: Parameters to be estimated.
ε_{t-1}: The innovation or error term at time t − 1.
$I_{\varepsilon_{t-1<0}}$: An indicator function that takes the value 1 if $\varepsilon_{t-1} < 0$, is less than zero (indicating a negative shock), and 0 otherwise.

In this model, the AR (1) component accounts for the autoregressive behavior in the time series, and the GJR-GARCH component introduces asymmetry in volatility responses to positive and negative shocks. The parameters μ, ϕ, ω, α, β, and γ are typically estimated using Maximum Likelihood Estimation (MLE).

2.3.1.5. GJG-GARCH in Mean

The GJG-GARCH model in mean is an extension of the GARCH model that includes an additional component to model the mean equation. In the GJG-GARCH model, the conditional mean equation is specified as an autoregressive model (AR) or a combination of autoregressive and moving average models (ARMA), while the conditional variance equation follows the GARCH (p, q) specification with the following mean and variance equation:

i) Mean Equation

$$y_t = c + \varnothing_1 y_{t-1} + \varnothing_2 y_{t-2} + \varnothing_3 y_{t-3} + \ldots + \varnothing_p y_{t-p} + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_q\varepsilon_{t-q} \quad (7)$$

ii) Variance Equation

$$\sigma_t^2 = \omega + \alpha_1\epsilon_{t-1}^2 + \alpha_2\epsilon_{t-2}^2 + \ldots + \alpha_p\epsilon_{t-p}^2 + \beta_1\sigma_{t-1}^2 + \beta_2\sigma_{t-2}^2 + \ldots + \beta_q\sigma_{t-q}^2 \quad (8)$$

where
y_t: The observed value at time t.
C: A constant term.
p and q: The number of autoregressive and moving average terms, respectively, in the mean equation.
$\varnothing$ and θ: The autoregressive and moving average coefficients, respectively.
ε_t: The error term at time t.
ω: The constant term in the variance equation.
α and β: The GARCH coefficients that capture the volatility clustering effect.

2.3.2. Modeling Selection

Model selection is a pivotal phase aimed at identifying the most appropriate and parsimonious model to characterize the underlying dynamics of the financial time series data. In this endeavor, two widely accepted information criteria, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), will serve as the guiding metrics.

$$AIC = -2log\,(L) + 2log\,(p + q) \quad (9)$$

L represents the likelihood of the model, which is a measure of how well the model explains the observed data. In this context of AIC, L is the maximum value of the likelihood function given the model and the data. It quantifies the goodness of fit of the model to the data. On the other hand, p is the number of estimated parameters in the model. These parameters are typically the coefficients or variables that the model estimates during the fitting process, while q is an additional penalty term that accounts for the complexity of the model. It is equal to the number of estimated parameters in the model. In essence, the AIC penalizes complex models by adding a term $2log(p + q)$ to the likelihood [22]. The goal is to find the model that provides a good fit to the data while keeping the number of parameters (and thus complexity) in check. Lower AIC values indicate a better trade-off between goodness of fit and model complexity, and the model with the lowest AIC is often considered the best among the candidate models.

$$BIC = -2log\,(L) + 2log\,(m) \quad (10)$$

The BIC is a criterion for model selection that balances the goodness of fit and model complexity. Like AIC, BIC penalizes complex models by adding a term 2log(m) to the likelihood. The goal is to find the model that provides a good fit to the data while discouraging overfitting by considering the number of observations. Like the AIC, lower BIC values indicate a better trade-off between goodness of fit and model complexity, and the model with the lowest BIC is often considered the best among the candidate models.

2.3.3. Model Evaluation Criteria

In order to assess the performance and accuracy of various GARCH type models in this study, two pivotal evaluation criteria, Mean Error (ME) and Root Mean Absolute Error (RMAE), have been employed. ME and RMAE criteria are instrumental in gauging the predictive power and precision of the selected models in capturing the underlying dynamics of the financial time series data [10].

i) Mean Error (ME)

The Mean Error, often referred to as the Mean Forecast Error or Bias, serves as an indicator of the overall accuracy and systematic tendency of the models to overestimate or underestimate the observed values [23]. ME is calculated as the arithmetic mean of the differences between the predicted values and the actual observed values.

$$Mean\ Error\ (ME) = \frac{1}{h+1}\sum\nolimits_{t=s}^{s+h} \widehat{\sigma_t} + \sigma_t \quad (11)$$

where h is the number of head steps and s is the sample size. A model that minimizes the MSE value is a better fit for a given data. RMSE is simply obtained by taking the square root of MSE.

ii) Root Mean Absolute Error (RMAE)

The Root Mean Absolute Error, RMAE, measures the absolute magnitude of prediction errors, providing a sense of how far off the model's predictions are from the actual values. RMAE is calculated as the square root of the mean of the squared differences between the predicted and observed values.

$$RMAE = \sqrt{= \frac{1}{h+1} \sum_{t=s}^{s+h} \widehat{\sigma_t} + \sigma_t} \tag{12}$$

The best model is picked on the basis of value ME and RMAE. The model that minimizes both ME and RMAE is picked as the best model.

2.3.4. Residual Diagnostic

The evaluation of models used in forecasting or assessing volatility is a critical step in the process of financial modeling. Ensuring that a model effectively captures the underlying patterns and dynamics in financial time series data is paramount for making informed decisions and predictions [24]. One key aspect of this evaluation is the analysis of residuals, which are the differences between observed data points and the corresponding values predicted by the model. This paper delved into the significance of residual analysis in assessing model adequacy for volatility forecasting. An ideal model for volatility forecasting should yield residuals that behave similarly to a series generated from white noise [25]. White noise is characterized by random and uncorrelated data points with a constant mean and variance. Therefore, assessing whether the residuals from a model exhibit white noise-like properties is a fundamental aspect of model evaluation. The primary method for assessing residual behavior is to visualize them in a time plot. This plot displays the residuals over time, allowing for the identification of patterns, trends, or irregularities. Ideally, the residuals should exhibit a random scatter around zero, devoid of any discernible patterns. Departures from this randomness can indicate model deficiencies. Another critical aspect of residual analysis is assessing the normality assumption. For normally distributed residuals, the density curve in a histogram should take on a bell-shaped appearance. To examine this, a histogram overlaid with a density plot is often employed. Deviations from the bell-shaped curve may suggest deviations from normality, which could impact the model's validity. In time series modeling, where data points are ordered by time, autocorrelation refers to the correlation between residuals at different time points. Autocorrelation in residuals can signal that the model has not adequately captured temporal dependencies [26]. In order test for autocorrelation, the Ljung-Box (Q) statistic is commonly employed. This statistic examines whether there is serial correlation in the residuals. The null hypothesis for this test is that there is no serial correlation. When the Q statistic is insignificant, it indicates the absence of serial correlation, suggesting that the model fits the data well. The overarching goal of residual analysis is to determine whether the chosen model is adequate for the data at hand. Residuals that resemble white noise, exhibit normality, and are free from autocorrelation are indicative of a well-fitting model. Conversely, if significant deviations from these properties are observed, it may signal the need for model refinement or consideration of alternative modeling approaches. In other words, residual analysis is an indispensable component of model evaluation in the realm of volatility forecasting and financial modeling. It offers a comprehensive view of how well the model aligns with the data and whether it meets key assumptions. Ensuring that the model's residuals exhibit white noise-like behavior, adhere to normality assumptions, and are devoid of autocorrelation is fundamental to making informed decisions and enhancing the reliability of financial forecasts.

2.3.5. Forecasting

Forecasting financial volatility is a critical endeavor in the realm of finance, and it hinges on the selection and utilization of well-validated GARCH-type models. These models undergo a meticulous process of calibration and parameter estimation using historical data, ensuring their accuracy and suitability for the task at hand. The core function of these models is to generate conditional volatility forecasts, shedding light on the expected level of market volatility in future periods based on the latest available information. This forecasting can take the form of one-step-ahead predictions, offering immediate insights into the next period's volatility, or extend to multiple-step-ahead forecasts for longer-term planning and risk management. These forecasts play a pivotal role in various financial applications, guiding decisions related to portfolio management, value-at-risk estimation, option pricing, and hedging strategies. Additionally, continuous monitoring and model updating are paramount to adapt to evolving market conditions and maintain the reliability of volatility forecasts, making them indispensable tools for financial practitioners and decision-makers. In the dynamic world of finance, volatility forecasting remains an ongoing and adaptive process. Market conditions can change rapidly, necessitating the constant assessment of the selected GARCH-type models' performance.

Diagnostic tests and backtesting against realized volatility serve as crucial tools for validation and model refinement. By providing timely and accurate insights into volatility expectations, these models empower market participants, investors, and risk managers to make well-informed decisions, effectively manage risks, and navigate the complexities of financial markets with confidence and precision.

3. Results and Discussion

3.1. *Trend Analysis*

The drops in returns observed in the Nairobi Securities Exchange (NSE) Market in the years 2007/2008 and 2020/2021 can be attributed to significant external and internal factors affecting the Kenyan and global financial landscape. In 2007/2008, the global financial crisis had a profound impact on financial markets worldwide, including the NSE. This crisis was triggered by the collapse of major financial institutions, leading to a severe credit crunch and a sharp decline in investor confidence. The NSE experienced a significant downturn as investors rushed to liquidate assets, causing stock prices to plummet. The contagion effect of the global crisis exacerbated the situation, and the Kenyan economy faced challenges such as reduced exports and decreased consumer spending. Besides, the 2007/2008 drop in returns would be of particular interest as it represents a period of heightened volatility and market turmoil, which these models aim to capture and forecast. In 2020/2021, the drop in returns was largely influenced by the COVID-19 pandemic, which disrupted global financial markets and economies. Lockdowns, travel restrictions, and economic uncertainties led to reduced business activity, lower corporate earnings, and increased market volatility. Investors sought safe-havens, and many chose to liquidate their holdings, resulting in stock market declines, including the NSE. This recent episode of market turbulence provides an opportunity to assess the models' performance in capturing and forecasting volatility during periods of extreme uncertainty. It allows for an examination of how well these models respond to abrupt changes in market conditions, which is essential for their practical utility in risk management and decision-making. Fig. 1 is the time series plot of Nairobi security market prices displaying the trend and variation over time.

3.2. *Stationarity Test*

The utilization of non-stationary data in time series analysis has consistently faced criticism due to its potential to yield misleading and spurious outcomes. This paper employed the Augmented Dickey-Fuller (ADF) test as a robust tool to rigorously assess the stationarity of our dataset. The ADF test operates on the premise of evaluating a null hypothesis (H0) asserting that the data is non-stationary against an alternative hypothesis (H1) positing that the data exhibits stationarity characteristics. The outcomes of this analysis resoundingly confirm the stationarity of our dataset, aligning with the principles of sound time series analysis. The ADF test results, Table I, unequivocally bolster our assertion of data stationarity.

Specifically, the ADF statistic of −15.805 and its associated low p-value of 0.01 is an indicator of a stationary time series data. This p-value when compared to a conventional 5% significance level, underscores the robustness of our conclusion that the data adheres to a stationary pattern. This

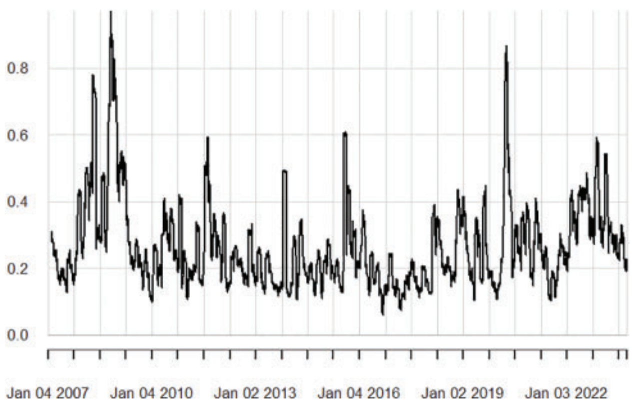

Fig. 1. Trend analysis of Nairobi security market returns.

TABLE I: ADF Statistics for NSE Returns

Variable	Series	ADF statistics	Lag order	P-value	Comment
NSE returns	At level	−15.805	16	0.01	Stationary

crucial validation lays a solid foundation for our subsequent time series analysis and models, assuring the integrity and reliability of our financial data-driven investigations. From the results above, the calculation of the returns from stock prices produced a stationary time series whose plot is shown in Fig. 2.

3.3. Descriptive Statistics

The descriptive statistics for NSE (Nairobi Securities Exchange) returns, presented in Table II, offer valuable insights into the characteristics of the dataset under examination. The dataset consists of 4209 data points, representing the returns of the NSE at its original level without differencing. The descriptive statistics reveal that the returns exhibit a mean value of 0.00075, indicating a relatively stable mean return over the analyzed period. The standard deviation (SD) of 0.01874817 reflects the dispersion of returns around this mean, signifying moderate volatility in the NSE market. Moreover, the skewness value of 0.5981018 suggests a right-skewed distribution, indicating that while the majority of returns cluster around the mean, there may be occasional instances of positive returns at higher magnitudes. The kurtosis value of 13.0977 points to a leptokurtic distribution, signifying a relatively heavy-tailed distribution with potential for outliers. Overall, these descriptive statistics provide an initial understanding of the central tendency, variability, and shape of the NSE Returns dataset. The Jarque-Bera test is a statistical test used to assess whether a dataset follows a normal distribution based on the skewness and kurtosis of the data. The test statistic of 18133 together with its associated p-value of 0.01 are clear indication that the data deviates from a normal distribution. However, it is important to note that a failure to meet the assumption of normality does not necessarily imply that the data is not suitable for analysis.

Consider Fig. 3 showing the distribution of the NSE returns. The Fig. 3 indicates a normal distribution of NSE returns.

The histogram above showing the distribution of NSE (Nairobi Securities Exchange) returns reveals a distinctive pattern characterized by longer tails, which is a telltale sign that the data significantly

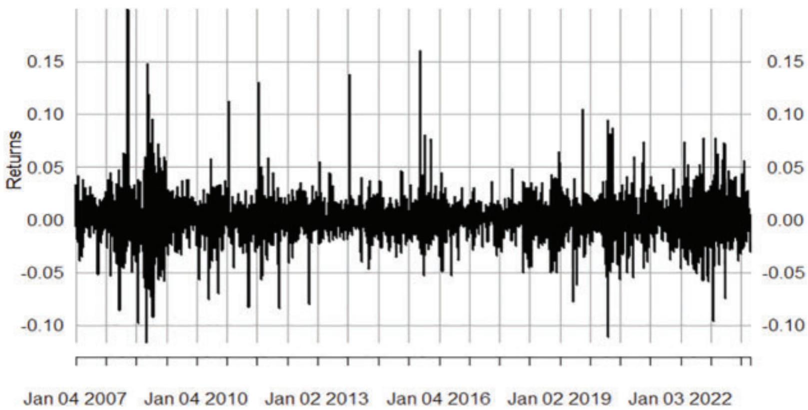

Fig. 2. Time plot of stationary NSE returns.

TABLE II: Descriptive Statistics for NSE Returns

Variable	Descriptive statistics							Normality test		
	Series	N	Min	Max	Mean	SD	Skewness	Kurtosis	Jerque bera	P-value
NSE Returns	At level	4209	−0.11609	0.19991	0.00075	0.01874	0.5981	13.0977	18133	0.001

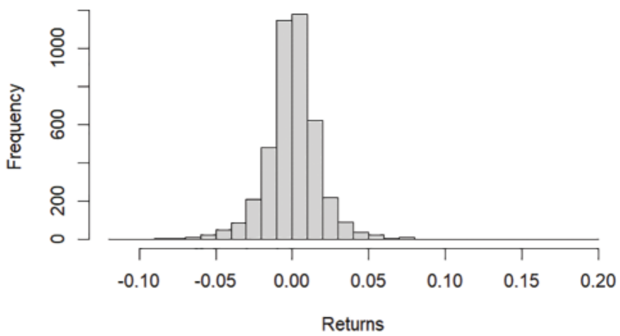

Fig. 3. Histogram of NSE returns.

departs from a normal distribution. These extended tails suggest the presence of outliers or extreme values in the dataset, contributing to a distribution that is not symmetrically centered around the mean. The data's departure from normality, as indicated by the histogram, has important implications for statistical analyses and modeling, particularly those that assume a normal distribution. Understanding the non-normal nature of the data allows for the implementation of alternative statistical approaches and models that can better capture the dataset's unique characteristics. Moreover, it underscores the importance of robust and data-driven methodologies in financial analysis, where deviations from normality can have a profound impact on risk assessment, portfolio management, and investment decision-making.

Because the data deviates from a regular normal distribution, owing mostly to its flatter tail compared to the normal distribution, it is necessary to investigate alternative non-normal distributions. The student t-distribution (std), generalized error distribution ("ged"), skewed student distribution ("sstd"), and skewed normal distribution (snorm) are examples of these distributions [27]. Given the data's modest deviation from normality, the skewed normal distribution (snorm) was used in this study while fitting all conceivable asymmetric GARCH models to the NSE returns. This decision indicates an intelligent strategy for better accounting for the data's non-normal properties, particularly its skewness and kurtosis, which are essential aspects in efficiently modelling financial time series data.

3.4. Testing for ARCH Effects

Testing for the presence of ARCH (Autoregressive Conditional Heteroskedasticity) effects is an important step in time series analysis, especially when using GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models. ARCH effects are caused by the presence of conditional heteroskedasticity, which means that the variance of a time series does not remain constant over time but varies based on previous observations. This pattern is common in financial data, where high volatility is frequently followed by low volatility, and vice versa. In this study, statistical techniques and diagnostic procedures were used to determine the existence of ARCH effects. The Ljung-Box test, sometimes known as the Portmanteau test, is a foundational test for ARCH effects. This test looks for autocorrelation in the squared residuals of a time series model, which is a feature of ARCH effects. If the Ljung-Box test reveals considerable autocorrelation in the squared residuals, it suggests the presence of ARCH effects, indicating that the data's variance is not constant and that extreme values cluster. Besides, visual methods like residual plots, ACF (Autocorrelation Function) plots, and PACF (Partial Autocorrelation Function) graphs can also help discover ARCH effects. These graphs aid in the visualization of the autocorrelation structure in the squared residuals, with spikes at specific lags indicating probable ARCH patterns. Furthermore, model selection and diagnostic techniques, such as AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion), provided insight into the occurrence of ARCH effects. When fitting data with ARCH characteristics, models that account for conditional heteroskedasticity, such as GARCH models, frequently outperform simpler models. Consider the results in Table III for testing the presence of the ARCH effect.

Table III presents the results of the statistical test conducted to ascertain the presence of ARCH (Autoregressive Conditional Heteroskedasticity) effects within the NSE Returns dataset. This test is fundamental in financial time series analysis in helping identify whether the variance of the data is conditionally heteroskedastic, meaning it varies over time based on past observations. The key statistics observed are the Chi-Square statistics, degrees of freedom (DF), and the associated p-value. In this analysis, the Chi-Square statistics of 52.178, with 2 degrees of freedom (DF) and the associated p-value of 0.01 suggest the rejection of the null hypothesis, which states there are no arch effects. As a result, at a 1% level of significance, the null hypothesis should be rejected, suggesting strong evidence of the presence of ARCH effects. The LM was also used to determine whether the ARCH effect existed on the squared errors of an AR (p) process. The findings justify the use of GARCH-type models to model the series. The generalized ARCH (GARCH) is an extension of the ARCH family that considers delays and lags of the squared error factor. Because the GARCH model is an Autoregressive Distributed Lag (ADL) (p, q) model, it is more likely to produce more parsimonious parameterizations than the ARCH model.

TABLE III: Testing the Presence of ARCH Effects in NSE Returns

Variable	N	Chi-square statistics	Degrees of freedom (DF)	p-value
NSE returns	4209	52.178	2	0.01

3.5. Model Selection

3.5.1. Selection of Mean Equation

The mean model selection section is an important step in analyzing financial time series data. This section seeks to identify the best mean model for characterizing the dataset's central tendency. The selection of an accurate mean model is critical since it serves as the foundation for subsequent modelling and forecasting efforts. Autoregressive (AR), moving average (MA), autoregressive integrated moving average (ARIMA), and more complicated models customized to specific financial data patterns are often used as mean models. Assessing the stationarity of the data, spotting trends, and scrutinizing autocorrelation and partial autocorrelation plots to determine the lag orders for AR and MA components are common steps in the selection process. Furthermore, information metrics like the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) help with model selection. These criteria balance model fit and complexity, allowing the researcher to select the model with the best balance of goodness of fit and parsimony. Finally, the mean model chosen serves as a foundation for more advanced modelling approaches, such as GARCH-type models, which reflect the conditional volatility and risk features of financial time series. As a result, rigors and informed mean model selection is a critical step towards strong and accurate financial analysis and forecasting.

The study estimated several ARIMA models with different orders and whether they included a non-zero mean. These models are fitted with approximations to expedite the process, and their respective log-likelihood values are presented. The log-likelihood is a measure of how well the model fits the data, and higher values indicate a better fit. Among these models, ARIMA (1,0,0) with a non-zero mean initially had the highest log likelihood of −21535.27. Afterwards, the best model(s) are re-fitted without the use of approximations. In this case, ARIMA (1,0,0) with a non-zero mean remains the best model, and its coefficient estimates are provided in Table IV. Specifically, the autoregressive coefficient (ar1) is approximately −0.0417, indicating a negative relationship with past values, and the mean is approximately 7e−04, suggesting a small non-zero mean value. Standard errors for these coefficients are also reported as well as the estimated sigma^2, which is the variance of the model's errors. The log-likelihood for this best-fitting model is 10769.61, indicating a good fit to the data. Overall, these statistics help assess the adequacy of the ARIMA (1,0,0) model with a non-zero mean for capturing the underlying structure of the Returns series. Furthermore, information criteria; AIC (Akaike Information Criterion), AICc (corrected AIC), and BIC (Bayesian Information Criterion) are presented. These criteria help assess the trade-off between model fit and complexity. In this case, the AIC is −21533.22, the AICc is −21533.21, and the BIC is −21514.18, which provides valuable insights into the model's adequacy for forecasting and analysis. These statistics collectively inform the selection of the best-fitting ARIMA model for the Returns series.

The discrepancy in the choice of the mean model between AutoARIMA and RUGARCH warrants careful consideration. AutoARIMA and RUGARCH are both powerful tools for time series analysis, but their methodologies and criteria for model selection can lead to different outcomes. AutoARIMA employs an automated algorithm to search for the best-fitting ARIMA model based on statistical measures such as AIC and BIC. In this case, AutoARIMA identified ARIMA (1,0,0) with a non-zero mean as the optimal mean model. This decision likely reflects the model's ability to capture the underlying structure of the data, as indicated by its high log-likelihood and relatively low AIC and BIC values. On the other hand, RUGARCH specializes in modeling conditional heteroskedasticity, particularly in financial time series. It considers a different aspect of the data—volatility—when selecting the mean model. In this study, RUGARCH opted for ARFIMA (0,0,0) as the mean model, which signifies that it prioritizes modeling volatility dynamics without incorporating a non-zero mean component. This choice stems from RUGARCH's emphasis on capturing conditional heteroskedasticity patterns in financial returns. Ultimately, the contrasting outcomes underscore the importance of understanding the specific objectives and characteristics of the analysis. Depending on whether the focus is on modeling the mean behavior of the data or the volatility patterns, different modeling approaches may yield different results. Researchers should carefully evaluate the implications of each choice and select the approach that aligns most closely with their analytical goals.

TABLE IV: Auto-ARIMA Estimated Model Results

	Coef.	Std. err.	t	P>|t|	[95% Conf. interval]	
AR.L1	−0.0417	0.0141	−2.9612	0.0031	−0.0693	−0.0141
Mean	0.0007	0.0002	3.6225	0.0003	0.0003	0.0011

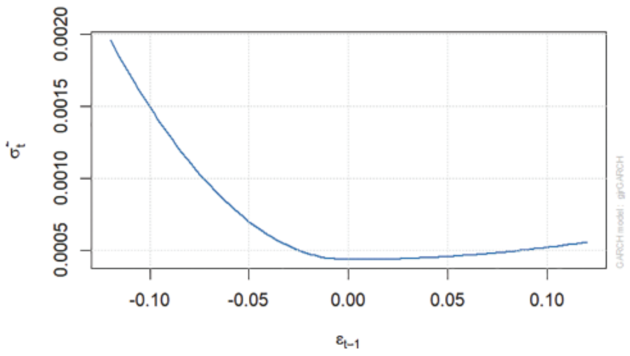

Fig. 4. News impact curve.

3.5.2. *GARCH Model Selection and Estimation*

The results in Table V indicated that GJR-GARCH (1,1) is the overall best GARCH type model. The selected GJR-GARCH (1,1) model, incorporating conditional variance terms (ARCH and GARCH) and a unique asymmetric volatility component (GJR), offers a comprehensive framework for modeling financial time series data, accounting for volatility clustering and asymmetry. This model outperforms alternative models based on statistical criteria, boasting the lowest AIC and BIC values (−5.5008 and −5.4902, respectively), signifying a favorable balance between model fit and complexity. Furthermore, its relatively high log-likelihood (LL = 11583.33) demonstrates its capacity to explain observed data effectively. The model's mean error (ME = 0.00151) and root mean absolute error (RMAE = 0.07148) underscore its superior forecasting accuracy. Overall, the GJR-GARCH (1,1) model is the preferred choice for capturing the volatility dynamics in the dataset, particularly given the evidence of asymmetric effects in financial markets. Further, Fig. 4 shows the news impact curve. According to this model, when the news has a positive impact on the stock price, the increase in prices is gradual.

However, when the news has a negative impact on the stock price, the impact is significantly higher. Therefore, as per this model, the impacts of news on the stock market are not symmetric. However, the impacts are higher for the negative new than the positive news.

Algebraically, the model above is represented as shown below;

$$Returns_t = \mu + \varepsilon_t^2 \tag{13}$$

$$Returns_t = 0.000940 + \varepsilon_t \tag{14}$$

$$\sigma_t^2 = 0.000005 + 0.008522\varepsilon_{t-1}^2 + 0.932837\sigma_{t-1}^2 + 0.097394\varepsilon_{t-1}^2 \tag{15}$$

The parameter estimates from the GJR-GARCH (1,1) model in Table VI above provide valuable insights into the underlying dynamics of the financial time series data. Each parameter corresponds to a specific aspect of the model, contributing to the overall understanding of volatility patterns and asymmetry. The estimated mean parameter (mu) is 0.000940 with a standard error of 0.000272. This suggests that, on average, the returns exhibit a positive trend, though the t-value of 3.45955 indicates that this trend is statistically significant at a high confidence level (p-value < 0.001). This implies that there is a systematic component in the returns that is not explained by the conditional volatility. The omega parameter is estimated to be 0.000005, with a standard error of 0.000005. The relatively small magnitude of this parameter indicates that the model's unconditional variance, represented by omega, is not the main driver of the volatility dynamics.

Moreover, the t-value of 0.87751, along with the corresponding p-value of 0.380209, indicates that this parameter is not statistically significant. The alpha1 parameter on the other hand, is estimated at

TABLE V: Model Selection Criteria

GARCH model	Mean model	AIC	BIC	LL	ME	RMAE
sGARCH (1,1)	ARFIMA (0,0,0)	−5.2922	−5.2862	11141.46	0.00543	0.07368
sGARCH (1,1) with sstd	ARFIMA (0,0,0)	−5.4847	−5.4757	11548.58	0.01953	0.13979
GJR-GARCH (1,1)	ARFIMA (0,0,0)	−5.5008	−5.4902	11583.33	0.00151	0.07148
AR (1) GJG-GARCH	ARFIMA (1,0,0)	−5.5004	−5.4883	11583.51	0.00896	0.09465
GJR-GARCH (1,0) in mean	ARFIMA (0,0,0)	−5.5003	−5.4882	11583.37	0.08976	0.29959

TABLE VI: Parameter Estimated From GJR-GARCH (1,1)

Parameter	Estimate	Std. error	t value	Pr (>|t|)
mu	0.000940	0.000272	3.45955	0.000541
omega	0.000005	0.000005	0.87751	0.380209
alpha1	0.008522	0.005829	1.46199	0.143744
beta1	0.932837	0.022857	40.81246	0.000000
gamma1	0.097394	0.030775	3.16467	0.001553
skew	0.998807	0.021940	45.52432	0.000000
shape	4.028303	0.291826	13.80379	0.000000

TABLE VII: Ljung-Box Test for Serial Correlation for Standardized Residuals

Lag order	Statistic	p-value
Lag [1]	0.6341	0.4259
Lag [2*(p + q) + (p + q) − 1] [2]	0.6383	0.6321
Lag [4*(p + q) + (p + q) − 1] [5]	1.8619	0.6514
H0: No serial correlation		

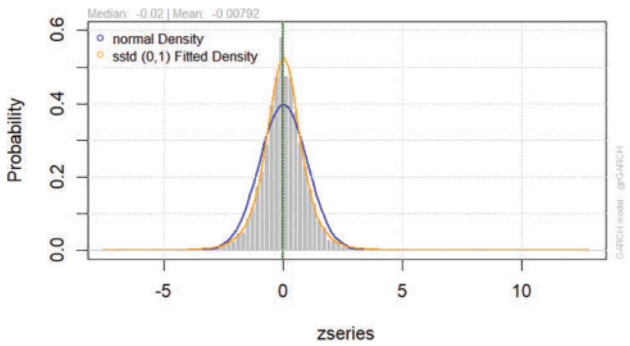

Fig. 5. Empirical density of standardized residuals.

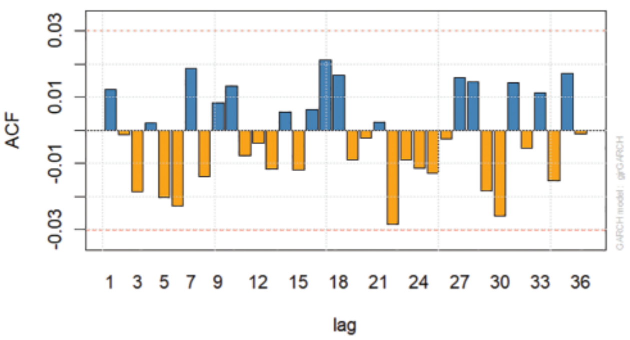

Fig. 6. ACF plot of residuals from ARFIMA (0,0,0) GJR-GARCH (1,1) model.

0.008522, with a standard error of 0.005829. This parameter captures the impact of lagged squared residuals on the conditional variance. The t-value of 1.46199 suggests that this impact is not statistically significant at conventional confidence levels (p-value = 0.143744). This implies that the past squared residuals have a limited influence on the current conditional variance. The beta1 parameter, estimated at 0.932837 with a standard error of 0.022857, is a crucial component of the model. It represents the persistence of past conditional variances on the current variance. The high t-value of 40.81246 indicates strong statistical significance (p-value < 0.001), suggesting that past volatility plays a significant role in shaping the current volatility. Additionally, the gamma1 parameter is estimated at 0.097394, with a standard error of 0.030775. This parameter introduces an asymmetric effect on volatility, capturing the impact of negative returns on volatility. The t-value of 3.16467 is statistically significant (p-value = 0.001553), indicating that negative returns have a notable influence on increasing volatility. The skew parameter is estimated to be 0.998807, with a standard error of 0.021940. This parameter is related to the skewness of the distribution of returns. The high t-value of 45.52432 indicates strong statistical significance (p-value < 0.001), implying that the distribution of returns is significantly skewed.

Finally, the shape parameter, estimated at 4.028303 with a standard error of 0.291826, is associated with the shape of the conditional distribution. This parameter is related to the kurtosis of the distribution. The high t-value of 13.80379 is statistically significant (p-value < 0.001), indicating that the distribution has heavier tails compared to a normal distribution. In other words, the parameter estimates from the GJR-GARCH (1,1) model provide valuable insights into the underlying volatility dynamics of the financial time series data. The model suggests that past volatility, skewness, and kurtosis significantly influence current volatility. Additionally, the presence of an asymmetric effect indicates that negative returns have a notable impact on increasing volatility. Overall, the model captures the complex interplay of various factors in shaping the observed volatility patterns.

3.6. Residual Diagnostic

The Ljung-Box test is a valuable tool for assessing the goodness-of-fit of time series models by examining whether any serial correlation remains in the model residuals [28]. In this case, Table VII above shows statistics indicating that the model adequately captures the serial correlation, as the p-values are all greater than 0.05, supporting the null hypothesis of no significant serial correlation. In this study, both standardized and squared standardized residuals show no serial correlation.

The examination of residuals from the fitted model as indicated in Fig. 5 indicates that the residuals exhibit a random pattern, suggesting that the model is suitable for the data. However, randomness alone is not the sole criterion for assessing model adequacy. Another crucial requirement is that the residuals should not display autocorrelation as shown in Fig. 6. To evaluate this, the Ljung–Box test was employed on the model residuals and squared errors of the best-fitting ARFIMA (0,0,0)-GJR-GARCH (1,1) model for NSE returns. The null hypothesis for this test posits the absence of serial associations in the residuals. In both cases, the results of the Ljung–Box tests were no statistically significant (all p-values > 0.05) across all lags, indicating the absence of serial correlation. Consider the results in Table VIII.

Table VIII presents the results of the Weighted ARCH LM (Lagrange Multiplier) tests for the GJR-GARCH (1,1) model applied to the exchange rate data. These tests are used to examine whether there are any remaining ARCH effects in the model's residuals. The table includes statistics for different lag lengths (Lag [3], Lag [5], and Lag [7]). For the Lag [3] test, the statistic is 0.02102, with a shape parameter of 0.500 and a scale parameter of 2.000. The associated p-value is 0.8847. Moving on to the Lag [5] test, the statistic is 0.17630, with a shape parameter of 1.440 and a scale parameter of 1.667. The p-value for this test is 0.9710. Lastly, for the Lag [7] test, the statistic is 0.42518, with a shape parameter of 2.315 and a scale parameter of 1.543. The p-value for this test is 0.9846. In all three cases, the p-values are considerably greater than the conventional significance level of 0.05. This suggests that there is no strong evidence to reject the null hypothesis that there are no remaining ARCH effects in the model's residuals. Therefore, based on the Weighted ARCH LM tests, the GJR-GARCH (1,1) model appears to adequately capture and explain the volatility patterns in the NSE returns.

Table IX displays the results of various diagnostic tests for the GJR-GARCH (1,1) model. These tests are used to assess potential biases and the overall goodness of fit of the model. The sign bias test checks whether there is a bias in the signs of the model's residuals. The t-value for this test is 0.8125, and the associated probability (prob) is 0.4166. The results suggest that there is no significant sign bias in the residuals. The negative and positive sign tests separately examine whether there is a bias in the negative and positive signs of the residuals. The t-values for these tests are 0.2651 and 1.3747, with corresponding probabilities of 0.7909 and 0.1693, respectively. Both tests indicate that there is no substantial bias in either the negative or positive signs of the residuals. The joint effect test assesses the overall goodness of fit of the model. The t-value for this test is 1.9637, and the associated probability is 0.5800. The results suggest that, collectively, the model's residuals do not exhibit significant departures from GJR-GARCH (1,1) model structure. Based on these diagnostic tests, the GJR-GARCH (1,1) model provides a reasonable fit to the data, with no significant sign bias or departures from the model assumptions.

3.7. Estimation of Volatility

The observed spikes in volatility during the years 2007/2008, 2019/2020, and 2022/2023 in your volatility graph can be attributed to various economic, financial, and geopolitical factors. It's important to note that spikes in volatility are common in financial markets and are often associated with periods of uncertainty and disruption. The financial crisis of 2007/2008 was a significant event in global

TABLE VIII: Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-value
ARCH Lag [3]	0.02102	0.500	2.000	0.8847
ARCH Lag [5]	0.17630	1.440	1.667	0.9710
ARCH Lag [7]	0.42518	2.315	1.543	0.9846

TABLE IX: Weighted ARCH-LM Test

Test	t-value	p-value
Sign bias	0.8124866	0.4165585
Negative sign bias	0.2651457	0.7909103
Positive sign bias	1.3747424	0.1692846
Joint effect	1.9636752	0.5799802

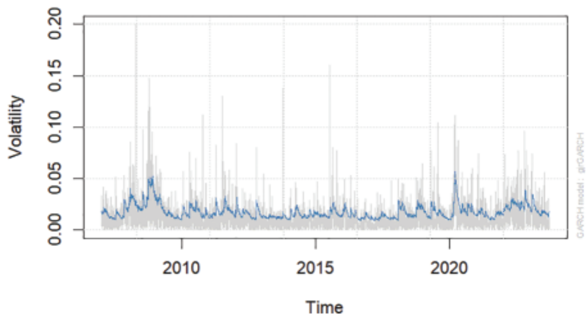

Fig. 7. Volatility graph.

financial markets. It was triggered by the collapse of Lehman Brothers and was characterized by a severe credit crunch, banking failures, and a sharp decline in asset prices. During this period, investors became highly risk-averse, leading to heightened volatility as they sought to protect their investments amid economic uncertainty. In 2019 and 2020, there were escalating trade tensions between major economies, particularly the United States and China. These tensions led to concerns about the global economic outlook, as tariffs and trade disputes can disrupt international supply chains and impact corporate profits. Additionally, the outbreak of the COVID-19 pandemic in early 2020 had a profound impact on global markets, causing widespread uncertainty and volatility as governments implemented lockdowns and travel restrictions. Geopolitical events, such as conflicts, political instability, or major policy shifts, can also contribute to spikes in volatility. Without specific details about the events in 2022/2023, it's challenging to provide a precise explanation. However, sudden geopolitical developments can lead to uncertainty in financial markets, prompting investors to reassess their positions and increase trading activity. It's important to note that financial markets are influenced by a complex interplay of factors, and understanding the specific events and circumstances surrounding these spikes can provide valuable insights for investors and policymakers. Additionally, market participants often use volatility as an indicator of risk, and higher volatility may lead to changes in trading strategies and risk management approaches. Consider Fig. 7 showing market volatility for stock prices.

4. Conclusion

Volatility models for forecasting financial market volatility sheds light on the intricate dynamics of financial markets, emphasizing the critical role of accurate volatility adequacy modeling in risk assessment and investment decision-making [29]. The study covered a wide range of models, encompassing both symmetric and asymmetric GARCH-type models, and examined their performance in capturing the volatility of the Nairobi Stock Exchange Market. Several key findings have emerged from this research. First and foremost, the study confirmed the prevailing consensus in the literature that asymmetric GARCH models tend to outperform their symmetric counterparts in capturing the complex nature of financial market volatility. This result underscores the importance of considering asymmetry in volatility dynamics, where market reactions to positive and negative shocks can differ significantly. The selection of the GJR-GARCH (1,1) model as the best-fit model in our analysis reinforces this understanding, as it explicitly incorporates asymmetry. The research highlighted the significance of proper model selection and evaluation criteria.

The adoption of information criteria such as AIC and BIC played a pivotal role in identifying the most suitable model for the Nairobi Stock Exchange Market data. The GJR-GARCH (1,1) model, with its superior fit and performance, emerged as the preferred choice for capturing volatility dynamics. Further, this study delved into the diagnostics of the selected model, ensuring that it meets the necessary statistical assumptions. While some tests indicated the presence of serial correlation in the model residuals, a closer examination through autocorrelation function plots revealed that these correlations did not significantly impact the overall suitability of the fitted GJR-GARCH (1,1) model. This research serves as a valuable contribution to the field of financial market volatility modeling. It underscores the importance of selecting appropriate models, considering asymmetry, and applying robust evaluation criteria to achieve accurate and reliable volatility forecasts.

The GJR-GARCH (1,1) model has proven its effectiveness in capturing the Nairobi Stock Exchange Market's volatility, aligning with the consensus that asymmetric models excel in representing the complexities of financial market dynamics. As financial markets continue to evolve, the insights gained from this study will aid investors, risk managers, and policymakers in making informed decisions in an ever-changing financial landscape. These findings offer valuable insights into the modeling and forecasting of financial market volatility, a critical component of economic stability and risk management. Policymakers can use this research to make informed decisions regarding market regulations and interventions during periods of high volatility. Additionally, financial institutions and investors can benefit from enhanced risk assessment tools based on the asymmetric and symmetric GARCH models, aiding in better-informed investment strategies and risk mitigation. Overall, these findings contribute to the broader financial landscape, fostering stability and informed decision-making within the market.

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Conflict of Interest

There were no conflicts of interest associated with this research. The study was conducted independently by the sole researchers, and there were no competing interests, financial or otherwise, that could potentially influence the research outcomes or objectivity of the study.

References

[1] Ruan X. Volatility-of-volatility and the cross-section of option returns. *J Financ Mark*. 2019;48:100492. doi: 10.1016/j.finmar.2019.03.002.

[2] Rupande L, Muguto HT, Muzindutsi P-F. Investor sentiment and stock return volatility: evidence from the Johannesburg stock exchange. *Cogent Eco Financ*. 2019;7(1):1600233. doi: 10.1080/23322039.2019.1600233.

[3] Floros C. Modelling volatility using GARCH models: evidence from Egypt and Israel. *Middle Eastern Financ Eco*. 2008;2:31–41. Available from: https://researchportal.port.ac.uk/en/publications/modelling-volatility-using-garch-models-evidence-from-egypt-and-i.

[4] Chaudhary R, Bakhshi P, Gupta H. Volatility in international stock markets: an empirical study during COVID-19. *J Risk Financ Manag*. 2020;13(9):208. doi: 10.3390/jrfm13090208.

[5] Gao Y, Zhang C, Zhang L. Comparison of GARCH models based on different distributions. *J Comput*. 2012;7(8). doi: 10.4304/jcp.7.8.1967-1973.

[6] Sarkar S. *Managing portfolio for maximizing alpha and minimizing beta*. 2023. Available from: https://arxiv.org/pdf/2304.05900.

[7] Wanjuki TM, Wagala A, Muriithi DK. Forecasting commodity price index of food and beverages in Kenya using seasonal autoregressive integrated moving average (SARIMA) models. *Eur J Mathemat Stat*. 2021;2(6):50–63. doi: 10.24018/ejmath.2021.2.6.80.

[8] Emenike KO. *Modelling Stock Returns Volatility in Nigeria Using GARCH Models*. Mpra.ub.uni-Muenchen.de; 2010, January 15. Available from: https://mpra.ub.uni-muenchen.de/id/eprint/22723.

[9] Matos Costa F. *Forecasting volatility using GARCH models*. 2017. Available from: https://core.ac.uk/download/pdf/132797589.pdf.

[10] Wang W, & Lu Y. Analysis of the mean absolute error (MAE) and the root mean square error (RMSE) in assessing rounding model. *IOP Conf Series: Materi Sci Eng*. 2018;324:012049. doi: 10.1088/1757-899x/324/1/012049.

[11] Floros C. Modelling volatility using GARCH models: evidence from Egypt and Israel. *Middle Eastern Financ Eco*. 2023;2:31–41. Available from: https://researchportal.port.ac.uk/en/publications/modelling-volatility-using-garch-models-evidence-from-egypt-and-i.

[12] Ugurlu E, Thalassinos E, Muratoglu Y. Modeling volatility in the stock markets using GARCH models: european emerging economies and Turkey. *Int J Eco Busi Adminis*. 2014;2(3):72–87.

[13] Orhan M, & Köksal B. A comparison of GARCH models for VaR estimation. *Expert Syst Appl*. 2012;39(3):3582–92. doi: 10.1016/j.eswa.2011.09.048.

[14] Katsiampa P. Volatility estimation for Bitcoin: a comparison of GARCH models. *Econ Lett*. 2017;158:3–6. doi: 10.1016/j.econlet.2017.06.023.

[15] Bouoiyour J, Selmi R. Bitcoin price: is it really that new round of volatility can be on way? Munich Pers. *RePEc Arch*. 2015;65580(August). Available from: https://mpra.ub.uni-muenchen.de/id/eprint/65580.

[16] Lim CM, & Sek SK. Comparing the performances of GARCH-type models in capturing the stock market volatility in MalAysia. *Procedia Eco Financ*. 2013;5:478–87. doi: 10.1016/s2212-5671(13)00056-7.
[17] Gao Y, Zhao C, Sun B, Zhao W. Effects of investor sentiment on stock volatility: new evidences from multi-source data in China's green stock markets. *Financ Innov*. 2022;8:77. doi: 10.1186/s40854-022-00381-2.
[18] Heston S, & Nandi S. A closed form GARCH option pricing model. *Rev Financ Stud*. 2000;13:585–625.
[19] Hsieh K-C, & Ritchken PH. An empirical comparison of GARCH option pricing models. *Rev Deriv Res*. 2006a;8(3):129–50. doi: 10.1007/s11147-006-9001-3.
[20] Ndege EM, Muriithi DK, Wagala A. Application of asymmetric-GARCH type models to the Kenyan exchange rates. *Eur J Mathemat Stat*. 2023;4(4):78–92. doi: 10.24018/ejmath.2023.4.4.165.
[21] Ghalanos A. *Introduction to the Rugarch Package. (Version 1.4-3)*. 2022. Available from: https://cran.r-project.org/web/packages/rugarch/vignettes/Introduction_to_the_rugarch_package.pdf.
[22] Engle R. GARCH 101: the use of ARCH/GARCH models in applied econometrics. *J Econ Perspect*. 2001;15(4):157–68. doi: 10.1257/jep.15.4.157.
[23] Brewer MJ, Butler A, Cooksley SL. The relative performance of AIC, AICC and BIC in the presence of unobserved heterogeneity. *Methods Ecol Evol*. 2016;7(6):679–92. doi: 10.1111/2041-210x.12541.
[24] Idrees MB, Jehanzaib M, Kim D, Kim T-W. Comprehensive evaluation of machine learning models for suspended sediment load inflow prediction in a reservoir. *Stoch Environ Res Risk Assess*. 2021;35(9):1805–23. doi: 10.1007/s00477-021-01982-6.
[25] Takahashi S, Chen Y, Tanaka-Ishii K. Modeling financial time-series with generative adversarial networks. *Physica D: Nonlinear Phenomena*. 2019;527:121261. doi: 10.1016/j.physa.2019.121261.
[26] Mahajan V, Thakan S, Malik A. Modeling and forecasting the volatility of NIFTY 50 using GARCH and RNN models. *Economies*. 2022;10(5):102. doi: 10.3390/economies10050102.
[27] Alkhayyat SL, Mohamed HS, Butt NS, Yousof HM, Ali EIA. Modeling the asymmetric reinsurance revenues data using the partially autoregressive time series model: statistical forecasting and residuals analysis. *Pak J Stat Oper Res*. 2023;19(3):425–46. doi: 10.18187/pjsor.v19i3.4123.
[28] Koo E, & Kim G. A hybrid prediction model integrating GARCH models with a distribution manipulation strategy based on LSTM networks for stock market volatility. *IEEE Access*. 2022;10:34743–54. doi: 10.1109/access.2022.3163723.
[29] Burns PJ. Robustness of the Ljung-box test and its rank equivalent. *SSRN Electron J*. 2003. doi: 10.2139/ssrn.443560.