Convex Structure in Generalized Fuzzy Metric Spaces

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Abstract— **In this paper, we introduce the concept of convex structure in generalized fuzzy metric spaces and proved common fixed point theorems for a pair of self-mappings under sufficient contractive type conditions.**

Key words — **Contraction mapping, Convex structure, Generalized fuzzy metric space, Metric space.**

I. INTRODUCTION

In Mathematics, the concept of fuzzy set was introduced by Zadeh [12]. It is a new way to represent vagueness in our daily life. In 1975, Kramosil and Michalek [4] introduced the concept of fuzzy metric spaces which opened a new way for further development of analysis in such spaces. George and Veeramani [1] modified the concept of fuzzy metric space. After that, several fixed point theorems proved in fuzzy metric spaces. In this paper, we introduce the concept of convex structure in generalized fuzzy metric spaces and obtain fixed point and common fixed point theorems for a pair of self-mappings under sufficient contractive type conditions with convex structure.

II. PRELIMINARIES

A. Definition $_b$2.1

Let $(X, G, *)$ be a G – fuzzy metric space and I =[0,1]. A continuous mapping $\triangle : X \times X \times I \to X$ is said to be a convex structure on X if for each $(x, y, k) \in X \times X \times I$ and $u \in X$, $G(u, \triangle(x, y, k), \triangle(x, y, k), t) \geq k G(u, x, x, t)$ b+ $(1 - k) G(u, y, y, t)$.

A space X together with a convex structure R is called a fuzzy convex metric space.

B. Definition 2.2

A sequence $\{x_n\}$ in a generalized fuzzy convex metric space $(X, G, *)$ is said to converge to $x \in X$ if $\lim_{n \to \infty} G(x, x, x_n, t) = 1$, for all $t > 0$.

C. Definition 2.3

Let $(X, G, *)$ be a generalized fuzzy convex metric space. A sequence $\{x_n\}$ is called Cauchy sequence if $\lim_{n \to \infty} G(x_{n+p}, x_{n+p}, x_n, t) = 1$, for all $t > 0$ and $p > 0$.

D. Definition 2.4

A generalized fuzzy convex metric space $(X, G, *)$ is said to be complete if every Cauchy sequence is convergent.

III. MAIN RESULTS

A. Theorem 3.1

Let $(X, G, *)$ be a generalized fuzzy convex metric space with $\triangle$ convex structure and let $\Gamma : X \to X$ be self mappings satisfying, $G(\Gamma x, \Gamma y, \Gamma y, kt) \geq G(x, \Gamma y, \Gamma y, t) * G(\Gamma x, y, y, t)$, for all x, $y \in X$ and $0 < k < 1$. Suppose that $\{x_n\}$ associated with Γ is defined by:

$$x_0 \in X \tag{1}$$

$$x_{n+1} = \triangle(\Gamma y_n, x_n, \alpha_n), n = 0, 1, 2\dots \tag{2}$$

$$y_n = \triangle(\Gamma x_n, x_n, \beta_n), n = 0, 1, 2\dots \tag{3}$$

where $0 \leq \alpha_n, \beta_n \leq 1$ and β_n is bounded away from zero. If $\{x_n\}$ converges to some point $x \in X$ then x is the fixed point of Γ.

B. Proof

Suppose $x_n \to x$.
$G(x_n, x_{n+1}, x_{n+1}, t) = G(x_n, \triangle(\Gamma y_n, x_n, \alpha_n), \triangle(\Gamma y_n, x_n, \alpha_n), t)$

$\geq \alpha_n G(x_n, \Gamma y_n, \Gamma y_n, t) + (1 - \alpha_n) G(x_n, x_n, x_n, t)$

$\geq \alpha_n G(x_n, \Gamma y_n, \Gamma y_n, t) + (1 - \alpha_n).$

Therefore $G(x_n, \Gamma y_n, \Gamma y_n, t) = 1$ and hence $\Gamma y_n \to x$.

$G(\Gamma x_n, y_n, y_n, t) = G(\Gamma x_n, \triangle(\Gamma x_n, x_n, \beta_n), \triangle(\Gamma x_n, x_n, \beta_n), t)$

$\geq \beta_n G(\Gamma x_n, \Gamma x_n, \Gamma x_n, t) + (1 - \beta_n) G(\Gamma x_n, x_n, x_n, t)$

$\geq \beta_n + (1 - \beta_n) G(\Gamma x_n, x_n, x_n, t) \tag{4}$

$G(\Gamma x_n, \Gamma y_n, \Gamma y_n, kt) \geq G(x_n, \Gamma y_n, \Gamma y_n, t) * G(\Gamma x_n, y_n, y_n, t)$

$\geq 1 * [\beta_n + (1 - \beta_n) G(\Gamma x_n, x_n, x_n, t)]$ by(4)
$\geq \beta_n + (1 - \beta_n) G(\Gamma x_n, x_n, x_n, t) \tag{5}$

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$G(\Gamma x_n, x_n, x_n, t) \geq G(\Gamma x_n, \Gamma y_n, \Gamma y_n, t/2) * G(\Gamma y_n, x_n, x_n, t/2)$

$\geq \beta_n + (1 - \beta_n) G(\Gamma x_n, x_n, x_n, t/2k)$.

It follows $G(\Gamma x_n, x_n, x_n, t) \geq 1$ and $Tx_n \to x$.

$G(\Gamma x_n, y_n, y_n, t) = G(\Gamma x, \triangle(\Gamma x_n, x_n, \beta_n), \triangle(\Gamma x_n, x_n, \beta_n), t)$

$\geq \beta_n G(\Gamma x_n, \Gamma x, \Gamma x, t) + (1 - \beta_n) G(x_n, \Gamma x, \Gamma x, t)$

$\geq \beta_n G(x, \Gamma x, \Gamma x, t) + (1 - \beta_n) G(x, \Gamma x, \Gamma x, t)$.

It follows $G(\Gamma x, x, x, t) = 1$. Thus $\Gamma x = x$. Hence x is a fixed point of Γ.

C. Theorem3.2

Let $(X, G, *)$ be a generalized fuzzy convex metric space and let $\Upsilon, \Gamma: X \to X$ be self mappings satisfying, $G(\Upsilon x, \Gamma y, \Gamma y, kt) \geq G(\Upsilon x, y, y, t) * G(x, \Gamma y, \Gamma y, t)$, for all x, y $\in$ X and $0 < k < 1$. Suppose that $\{x_n\}$ associated with S and Γ is defined by:

$x_0 \in X$ (6)

$x_{n+1} = \triangle(\Gamma y_n, x_n, \alpha_n)$, n = 0, 1, 2…   (7)

$y_n = \triangle(\Upsilon x_n, x_n, \beta_n)$, n = 0, 1, 2…   (8)

where $0 \leq \alpha_n, \beta_n \leq 1$ and β_n is bounded away from zero. If $\{x_n\}$ converges to some point x $\in$ X then x is the common fixed point of Γ and Υ.

D. Proof

Suppose $x_n \to x$.

$G(x_n, x_{n+1}, x_{n+1}, t) = G(x_n, \triangle(\Gamma y_n, x_n, \alpha_n), \triangle(\Gamma y_n, x_n, \alpha_n), t)$

$\geq \alpha_n G(x_n, \Gamma y_n, \Gamma y_n, t) + (1 - \alpha_n) G(x_n, x_n, x_n, t)$

$\geq \alpha_n G(x_n, \Gamma y_n, \Gamma y_n, t) + (1 - \alpha_n)$.

Therefore, $G(x_n, \Gamma y_n, \Gamma y_n, t) = 1$ and hence $\Gamma y_n \to x$. Therefore, by (7) and (8),

$G(x_n, y_n, y_n, t) = G(x_n, \triangle(\Upsilon x_n, x_n, \beta_n), \triangle(\Upsilon x_n, x_n, \beta_n), t)$

$\geq \beta_n G(x_n, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n) G(x_n, x_n, x_n, t)$

$\geq \beta_n G(x_n, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n)$ (9)

$G(\Upsilon x_n, y_n, y_n, t) = G(\Upsilon x_n, \triangle(\Upsilon x_n, x_n, \beta_n), \triangle(\Upsilon x_n, x_n, \beta_n), t)$

$\geq \beta_n G(\Upsilon x_n, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n) G(\Upsilon x_n, x_n, x_n, t)$

$\geq \beta_n + (1 - \beta_n) G(\Upsilon x_n, x_n, x_n, t)$ (10)

$G(\Upsilon x_n, \Gamma y_n, \Gamma y_n, kt) \geq G(\Upsilon x_n, y_n, y_n, t) * G(x_n, \Gamma y_n, \Gamma y_n, t)$

$\geq [\beta_n + (1 - \beta_n) G(\Upsilon x_n, x_n, x_n, t)] * 1$ (11)

$\geq \beta_n + (1-\beta_n) G(\Upsilon x_n, x_n, x_n, t)$ by (10)

$G(\Upsilon x_n, x_n, x_n, t) \geq G(\Upsilon x_n, \Gamma y_n, \Gamma y_n, t/2) * G(\Gamma y_n, x_n, x_n, t/2)$

$\geq \beta_n G(x_n, \Upsilon x_n, \Upsilon x_n, t/2k) * (1 - \beta_n)$.

It follows $G(\Upsilon x_n, x_n, x_n, t) \geq 1$ and $\Upsilon x_n \to x$. Also, $y_n \to x$.

$G(\Gamma x, y_n, y_n, t) = G(\Gamma x, \triangle(\Upsilon x_n, x_n, \beta_n), \triangle(\Upsilon x_n, x_n, \beta_n), t)$

$\geq \beta_n G(\Upsilon x_n, \Gamma x, \Gamma x, t) + (1 - \beta_n) G(x_n, \Gamma x, \Gamma x, t)$

$\geq \beta_n G(x, \Gamma x, \Gamma x, t) + (1 - \beta_n) G(x, \Gamma x, \Gamma x, t)$.

It follows $G(\Gamma x, x, x, t) = 1$. Thus $\Gamma x = x$.

$G(\Upsilon x_n, y_n, y_n, t) = G(\Upsilon x, \triangle(\Upsilon x_n, x_n, \beta_n), \triangle(\Upsilon x_n, x_n, \beta_n), t)$

$\geq \beta_n G(\Upsilon x, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n) G(\Upsilon x, x_n, x_n, t)$

$\geq \beta_n G(\Upsilon x, x, x, t) + (1 - \beta_n) G(\Upsilon x, x, x, t)$.

Thus, $\Gamma x = \Upsilon x = x$. Hence x is the common fixed point of Γ and Υ.

E. Theorem 3.3

Let $(X, G, *)$ be a generalized fuzzy convex metric space and let $\Upsilon, \Gamma: X \to X$ be self mappings satisfying, $G(\Upsilon x, \Gamma y, \Gamma y, kt) \geq G(x, y, y, t)$, for all x, y $\in$ X and $0 < k < 1$. Suppose that $\{x_n\}$ associated with Υ and Γ is defined by:

$x_0 \in X$ (12)

$x_{n+1} = \triangle(\Gamma y_n, x_n, \alpha_n)$, n = 0, 1, 2…   (13)

$y_n = \triangle(\Upsilon x_n, x_n, \beta_n)$, n = 0, 1, 2…   (14)

where $0 \leq \alpha_n, \beta_n \leq 1$ and β_n is bounded away from zero.

If $\{x_n\}$ converges to some point x $\in$ X then x is the common fixed point of Γ and Υ.

F. Proof

Suppose $x_n \to x$.

$G(x_n, x_{n+1}, x_{n+1}, t) = G(x_n, \triangle(\Gamma y_n, x_n, \alpha_n), \triangle(\Gamma y_n, x_n, \alpha_n), t)$

$\geq \alpha_n G(x_n, \Gamma y_n, \Gamma y_n, t) + (1 - \alpha_n)$

$\geq \alpha_n G(x_n, \Gamma y_n, \Gamma y_n, t) + (1 - \alpha_n)$.

Therefore, $G(x_n, \Gamma y_n, \Gamma y_n, t) = 1$ and hence $\Gamma y_n \to x$.

$G(x_n, y_n, y_n, t) = G(x_n, \triangle(\Upsilon x_n, x_n, \beta_n), \triangle(\Upsilon x_n, x_n, \beta_n), t)$

$\geq \beta_n G(x_n, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n) G(x_n, x_n, x_n, t)$ (15)

$\geq \beta_n G(x_n, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n)$.

$G(\Upsilon x_n, y_n, y_n, t) = G(\Upsilon x_n, \triangle(\Upsilon x_n, x_n, \beta_n), \triangle(\Upsilon x_n, x_n, \beta_n), t)$

$\geq \beta_n G(\Upsilon x_n, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n) G(\Upsilon x_n, x_n, x_n, t)$ (16)

$\geq \beta_n + (1 - \beta_n)\, G(\Upsilon x_n, x_n, x_n, t).$

$G(\Upsilon x_n, \Gamma y_n, \Gamma y_n, kt) \geq G(x_n, y_n, y_n, t)$

$\geq \beta_n\, G(x_n, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n) \quad$ by (15)

$G(\Upsilon x_n, x_n, x_n, t) \geq G(\Upsilon x_n, \Gamma y_n, \Gamma y_n, t/2) * G(Ty_n, x_n, x_n, t/2)$

$\geq \beta_n G(x_n, \Upsilon x_n, \Upsilon x_n, t/2k) + (1 - \beta_n).$

It follows $G(\Upsilon x_n, x_n, x_n, t) \geq 1$ and $\Upsilon x_n \to x$. Also $y_n \to x$. Now, $G(\Gamma x, y_n, y_n, t) = G(\Gamma x, \triangle(\Upsilon x_n, x_n, \beta_n), \triangle(\Upsilon x_n, x_n, \beta_n), t)$

$\geq \beta_n G(\Upsilon x_n, \Gamma x, \Gamma x, t) + (1 - \beta_n)\, G(x_n, \Gamma x, \Gamma x, t)$
$\geq \beta_n G(x, \Gamma x, \Gamma x, t) + (1 - \beta_n)\, G(x, \Gamma x, \Gamma x, t).$

It follows $G(\Gamma x, x, x, t) = 1$. Thus $\Gamma x = x$.
$G(\Upsilon x, y_n, y_n, t) = G(\Upsilon x, \triangle(\Upsilon x_n, x_n, \beta_n), \triangle(\Upsilon x_n, x_n, \beta_n), t)$

$\geq \beta_n G(\Upsilon x, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n)\, G(\Upsilon x, x_n, x_n, t)$

$\geq \beta_n G(\Upsilon x, x, x, t) + (1 - \beta_n)\, G(\Upsilon x, x, x, t).$

Thus, $\Gamma x = \Upsilon x = x$. Hence x is the common fixed point of Γ and Υ.

G. Theorem3.4

Let $(X, G, *)$ be a generalized fuzzy convex metric space and let $\Upsilon, \Gamma: X \to X$ be self mappings satisfying, $G(\Gamma x, \Gamma y, \Gamma y, kt) \geq G(\Upsilon x, \Upsilon y, \Upsilon y, t)$, for all x, y $\in$ X and $0 < k < 1$. Suppose that $\{x_n\}$ associated with Υ and Γ is defined by

$x_0 \in X \hfill (17)$

$x_{n+1} = \triangle(\Gamma y_n, x_n, \alpha_n),\ n = 0, 1, 2\ldots \hfill (18)$

$y_n = \triangle(\Upsilon x_n, x_n, \beta_n),\ n = 0, 1, 2\ldots \hfill (19)$

where $0 \leq \alpha_n, \beta_n \leq 1$ and β_n is bounded away from zero. If $\{x_n\}$ converges to some point x $\in$ X then x is the common fixed point of Γ and Υ.

H. Proof
Suppose $x_n \to x$.

$G(x_n, x_{n+1}, x_{n+1}, t) = G(x_n, \triangle(\Gamma y_n, x_n, \alpha_n), \triangle(\Gamma y_n, x_n, \alpha_n), t)$

$\geq \alpha_n G(x_n, \Gamma y_n, \Gamma y_n, t) + (1 - \alpha_n)\, G(x_n, x_n, x_n, t)$
$\geq \alpha_n G(x_n, \Gamma y_n, \Gamma y_n, t) + (1 - \alpha_n).$

Therefore, $G(x_n, \Gamma y_n, \Gamma y_n, t) = 1$ and hence $\Gamma y_n \to x$.

$G(x_n, y_n, y_n, t) = G(x_n, \triangle(\Upsilon x_n, x_n, \beta_n), \triangle(\Upsilon x_n, x_n, \beta_n), t)$

$\geq \beta_n G(x_n, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n)\, G(x_n, x_n, x_n, t) \hfill (20)$

$\geq \beta_n G(x_n, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n)$ and

$G(\Upsilon x_n, y_n, y_n, t) = G(\Upsilon x_n, \triangle(\Upsilon x_n, x_n, \beta_n), \triangle(\Upsilon x_n, x_n, \beta_n), t)$

$\geq \beta_n G(\Upsilon x_n, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n)\, G(\Upsilon x_n, x_n, x_n, t) \hfill (21)$

$\geq \beta_n + (1 - \beta_n)\, G(\Upsilon x_n, x_n, x_n, t).$

$G(\Upsilon x_n, \Gamma y_n, \Gamma y_n, kt) \geq G(x_n, y_n, y_n, t)$

$\geq \beta_n G(x_n, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n) \quad$ by (20)

$G(\Upsilon x_n, x_n, x_n, t) \geq G(\Upsilon x_n, \Gamma y_n, \Gamma y_n, t/2) * G(\Gamma y_n, x_n, x_n, t/2)$

$\geq \beta_n G(x_n, \Upsilon x_n, \Upsilon x_n, t/2k) + (1 - \beta_n).$

It follows $G(\Upsilon x_n, x_n, x_n, t) \geq 1$ and $\Upsilon x_n \to x$. Also, $y_n \to x$.

$G(\Gamma x, y_n, y_n, t) = G(\Gamma x, \triangle(\Upsilon x_n, x_n, \beta_n), \triangle(\Upsilon x_n, x_n, \beta_n), t)$

$\geq \beta_n G(\Upsilon x_n, \Gamma x, \Gamma x, t) + (1 - \beta_n)\, G(x_n, \Gamma x, \Gamma x, t)$

$\geq \beta_n G(x, \Gamma x, \Gamma x, t) + (1 - \beta_n)\, G(x, \Gamma x, \Gamma x, t).$

It follows $G(\Gamma x, x, x, t) = 1$. Thus $\Gamma x = x$.

$G(\Upsilon x, y_n, y_n, t) = G(\Upsilon x, \triangle(\Upsilon x_n, x_n, \beta_n), \triangle(\Upsilon x_n, x_n, \beta_n), t)$

$\geq \beta_n G(\Upsilon x, \Upsilon x_n, \Upsilon x_n, t) + (1 - \beta_n)\, G(\Upsilon x, x_n, x_n, t)$

$\geq \beta_n G(\Upsilon x, x, x, t) + (1 - \beta_n)\, G(\Upsilon x, x, x, t).$

Thus, $\Gamma x = \Upsilon x = x$. Hence x is the common fixed point of Γ and Υ.

I. Definition 3.5

Let $(X, G, *)$ be a complete generalized fuzzy convex metric space and let $\Gamma: X \to X$ be continuous mapping Γ is a quasi-non expansive if $G(\Gamma x, x, x, t) \geq G(x, x, x, t)$, for all x $\in$ X and x is a fixed point of Γ.

J. Theorem 3.6

Let $(X, G, *)$ be a complete generalized fuzzy convex metric space and let $\Upsilon, \Gamma: X \to X$ be quasi-non expansive mappings. Suppose that $\{x_n\}$ associated with Υ and Γ is defined by:

$x_0 \in X \hfill (22)$

$x_{n+1} = \triangle(\Gamma y_n, x_n, \alpha_n),\ n = 0, 1, 2\ldots \hfill (23)$

$y_n = \triangle(\Upsilon x_n, x_n, \beta_n),\ n = 0, 1, 2\ldots \hfill (24)$

where $0 \leq \alpha_n, \beta_n \leq 1$ and $\{\alpha_n\}$ is bounded away from zero. If $\{x_n\}$ converges to some point x $\in$ X, then x is the common fixed point of Γ and Υ.

K. Proof
Let x $\in$ F(Γ) $\cap$ F(Υ). Then:

$G(x_{n+1}, x, x, t) = G(\triangle(\Gamma y_n, x_n, \alpha_n), x, x, t)$

$\geq \alpha_n G(\Gamma y_n, x, x, t) + (1 - \alpha_n)\, G(x_n, x, x, t)$

$\geq \alpha_n G(y_n, x, x, t) + (1 - \alpha_n)\, G(x_n, x, x, t)$

$\geq \alpha_n G(\triangle(\Upsilon x_n, x_n, \beta_n), x, x, t) + (1 - \alpha_n)\, G(x_n, x, x, t)$

$\geq \alpha_n \beta_n \, G(\Upsilon x_n, x, x, t) + \alpha_n \, (1 - \beta_n) \, G(x_n, x, x, t)$

$\qquad\qquad + (1 - \alpha_n) G(x_n, x, x, t)$

$\geq [\alpha_n \beta_n + \alpha_n \, (1 - \beta_n) + (1 - \alpha_n)] \, G(x_n, x, x, t)$

$\geq G(x_n, x, x, t).$

Thus, $G(x_{n+1}, x, x, t) \geq G(x_n, x, x, t) \geq G(x_{n-1}, x, x, t) \geq \ldots$
This implies $G(x_n, x, x, t) \geq 1$. Hence $\{x_n\}$ converges to x. The following result is a special case of the above theorem.

L. Corollary 3.7

Let $(X, G, *)$ be a complete generalized fuzzy convex metric space and let $\Gamma: X \to X$ be quasi-non expansive mappings. The sequence $\{x_n\}$ is defined by:

$x_0 \in X$ (25)

and
$x_{n+1} = \triangle(\Gamma y_n, x_n, \alpha_n)$ (26)

$y_n = \triangle(\Gamma x_n, x_n, \beta_n), \, n = 0, 1, 2, \ldots$ (27)

where $0 \leq \alpha_n, \beta_n \leq 1$. Then $\{x_n\}$ converges to the fixed point of Γ.

M. Proof

The proof of the corollary immediately follows by putting $\Upsilon = \Gamma$ in the previous theorem.

IV. CONCLUSION

In this paper, we have explored the concept of convex structure in generalized fuzzy metric spaces and proved common fixed point theorems for a pair of self-mappings under sufficient contractive type conditions.

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