

The Impact of Nanofluid Flow in a Rhombus-Shaped Porous Enclosure on a Constant Heated Obstacle

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ABSTRACT

In this study, the focus is on investigating the characteristic of a nanofluid consisting of water as the base fluid and Copper nanoparticles. The analysis examines the impact of this nanofluid flow on a heated obstacle placed in a rhombus-shaped enclosure filled with porous media. The driving force behind the mixed convection is assumed to be the temperature difference created by the moving walls, where the top and bottom walls have a cold-to-hot flow in opposite directions. To obtain numerical solutions, the Galerkin weighted residual method is employed. The study explores the effects of the Darcy number and Richardson number on various aspects, including streamlines, isotherms, dimensionless temperature, velocity profiles, average Nusselt numbers, and average fluid temperature. These effects are visualized through graphical representations. The findings reveal that both the Richardson number and Darcy number significantly influence the streamlines and isotherms within the system. Furthermore, the Darcy number proves to be an important control parameter for heat transfer in fluid flow through a porous medium in the enclosure. A heat transfer correlation for the average Nusselt number is provided, considering different Darcy numbers and Richardson numbers. To ensure the validity of the research, a comparison between the obtained results and previously published findings is conducted, showing a favorable agreement between them.

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1. INTRODUCTION

Studying the flow of nanofluids within a rhombus-shaped enclosure filled with porous media and featuring a constant heated obstacle is a topic of great relevance in today's world. Such enclosures, when filled with electrically conducting fluids, play a central role in various engineering and geophysical systems. The outcomes of this research have the potential to be applied in a range of practical applications including escalators, lifts, ventilation systems, heating and cooling technologies, building climate control, heat exchangers, solar collectors, as well as in astrophysics and biology. In-depth discussions on these application areas can be found in the comprehensive reviews by Nield and Bejan [1] and Ingham and Pop [2]. Improving heat transfer in cavities has become a critical concern in both industrial and energy sectors. Over the past years, researchers have conducted extensive investigations using experimental, analytical, and numerical methods to enhance heat transfer within cavities. One notable study by Xuan and Li [3] focused on the utilization of nanofluids to enhance heat transfer. By introducing CuO nanoparticles into the base fluid, they discovered that nanofluids have significant potential in improving the heat transfer process. In order to meet the growing demands of modern technologies, such as chemical production, power stations, and microelectronics, there is a need to develop fluids that can effectively facilitate heat exchange. A promising class of fluids in this regard



is nanofluids. These nanofluids are a type of heat transfer fluid that contains a small amount of nanoparticles suspended in a base fluid. It has been extensively studied by numerous researchers [4]–[6] that the addition of nanoparticles to the base fluid, even at low volume fractions (1%–5%), can significantly enhance the thermal conductivity by approximately 20%.

In a related study, Rahman *et al.* [7] investigated the effects of natural convection on heat and mass transfer within a curvilinear triangular cavity. Additionally, Chamkha and Naser [8] conducted research on double-diffusive convection occurring in an inclined porous enclosure characterized by opposing temperature and concentration gradients. Finite volume method was employed by Teamah [9] to conduct numerical simulations of double-diffusive natural convective flow in a rectangular enclosure. Their study considered the influence of a magnetic field and heat source on the flow. Basak *et al.* [10] examined the characteristics of lid-driven mixed convection in a porous square cavity. They found that at Reynolds number of 100, the dominant form of heat transfer was forced convection when the left wall linearly heated and the right wall was cooled. Boulahia *et al.* [11] investigated the combined convection of nanofluids within a square enclosure with a triangular heated block. Their findings indicated that the heat transfer rate increased with higher volume fractions of nanoparticles and lower Richardson numbers. Furthermore, Dawood and Teamah [12] conducted a study on hydro-magnetic mixed convection double-diffusive flow in a lid-driven square cavity. Oztop *et al.* [13] investigated MHD mixed convection in a lid-driven cavity with a heater located at one of the corners. Iwastu *et al.* [14] carried out numerical simulations on mixed convection within a driven cavity featuring a stable vertical temperature gradient. Munshi *et al.* [15] analyzed the numerical simulation of mixed convection heat transfer using nanofluids in a lid-driven square enclosure filled with a porous medium. Additionally, Munshi *et al.* [16] investigated hydrodynamic mixed convection in a lid-driven square cavity, which included an elliptic-shaped heated block with a corner heater. Saha *et al.* [17] conducted research on mixed convection heat transfer in a lid-driven cavity with a wavy bottom surface. Maxwell-Garnett [18] writes colures in metal glasses and in metallic films. Brinkman [19] analyzed the viscosity of concentrated suspensions and solution. Zienktewicz *et al.* [20] analyzed the finite element method.

The problem of mixed convection heat transfer in a lid-driven enclosure with various aspects has received limited attention in existing literature. Therefore, this study aims to address this research gap by proposing the impact of nanofluid flow in a rhombus-shaped porous on a constant heated obstacle. The primary objective of the present study is to simulate and analyze the aforementioned scenario.

2. PHYSICAL CONFIGURATION

Fig. 1 illustrates a schematic representation of the simulation setup for studying the effects of a constant heated obstacle on nanofluid flow in a rhombus-shaped enclosure filled with porous media. The top wall of the enclosure is characterized by a cold temperature filled with porous media. The top wall of the enclosure is characterized by a cold temperature and the lid moves from left to right, while the bottom wall also has a cold temperature but with the lid moving in the opposite direction, from right to left. The left and right walls of the enclosure are considered adiabatic, and within the enclosure, there exists a constant heated obstacle.

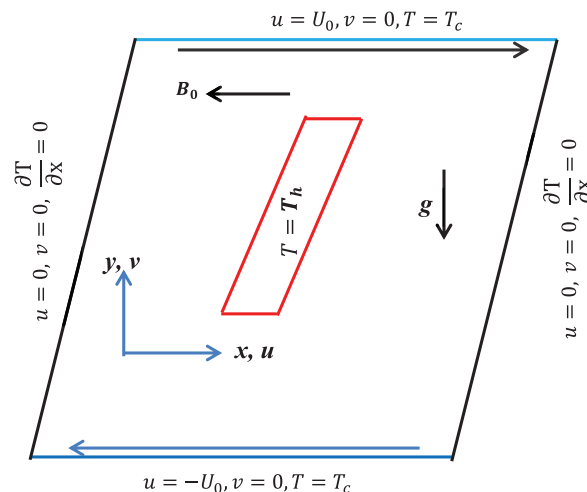


Fig. 1. Schematic configurations of the present physical model.

3. MATHEMATICAL FORMULATION

The given mathematical equations can be expressed as follows:

$$\underline{\nabla} \cdot \underline{q} = 0 \quad (1)$$

$$\rho_{nf} (\underline{q} \cdot \underline{\nabla} u) = -\frac{\partial p}{\partial x} + \mu_{nf} \nabla^2 u - \frac{(\rho\beta)_{nf}}{k} v u \quad (2)$$

$$\rho_{nf} (\underline{q} \cdot \underline{\nabla} v) = -\frac{\partial p}{\partial y} + \mu_{nf} \nabla^2 v - \frac{(\rho\beta)_{nf}}{k} v v + (\rho\beta)_{nf} g\beta (T - T_c) - \sigma_{nf} B_0^2 v \quad (3)$$

$$\underline{q} \cdot \underline{\nabla} T = \alpha_{nf} \nabla^2 T \quad (4)$$

$$\rho_{nf} = (1 - \varphi) \rho_f + \varphi \rho_s \quad (5)$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \quad (6)$$

$$(\rho c_p)_{nf} = (1 - \varphi) (\rho c_p)_f + \varphi (\rho c_p)_s \quad (7)$$

$$(\rho\beta)_{nf} = (1 - \varphi) (\rho\beta)_f + \varphi (\rho\beta)_s \quad (8)$$

Brinkman [19] as follows:

$$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}} \quad (9)$$

Maxwell (by the Maxwell model [18]) as follows:

$$k_{nf} = \frac{k_s + 2k_f - 2\varphi (k_f - k_s)}{k_s + 2k_f + \varphi (k_f - k_s)} \times k_f \quad (10)$$

The effective dynamic viscosity of the Copper-water nanofluid is calculated according to the Brinkman model [19] (Table I).

3.1. Boundary Conditions (Dimensional)

The dimensional boundary conditions for the solving the governing (1)–(4) are presented in Table II.

TABLE I: THERMO-PHYSICAL PROPERTIES OF WATER AND COPPER NANOPARTICLES [15]

Physical properties	Water	Copper
C_p	4179	385
ρ	997.1	8933
K	0.613	401
β	21×10^{-5}	1.67×10^{-5}
μ	0.855×10^{-3}	–

TABLE II: BOUNDARY CONDITIONS [15]

Walls	Boundary conditions
Top wall	$u = U_0, v = 0, T = T_c$
Left wall	$u = 0, v = 0, \partial T / \partial x = 0$
Right wall	$u = 0, v = 0, \partial T / \partial x = 0$
Bottom wall	$u = -U_0, v = 0, T = T_c$
Constant heated obstacle	$u = 0, v = 0, T = T_h$

3.2. Dimensionless Analysis

Using the following dimensionless parameters, the governing equations can be converted to the dimensionless forms:

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{u}{U_0 \alpha_f}, V = \frac{v}{U_0 \alpha_f}, P = \frac{p}{\rho_{nf} U_0^2}, \theta = \frac{T - T_c}{T_h - T_c}$$

The dimensionless forms of the governing equations expressed are:

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \frac{\mu_{nf}}{\rho_{nf} \cdot \gamma_f} \frac{1}{(1-\varphi)^{2.5}} \nabla^2 U - \frac{(\rho\beta)_{nf}}{\rho_{nf} \beta_f} \frac{1}{Re Da} U \quad (11)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \frac{\mu_{nf}}{\rho_{nf} \cdot \gamma_f} \frac{1}{(1-\varphi)^{2.5}} \nabla^2 U - \frac{(\rho\beta)_{nf}}{\rho_{nf} \beta_f} \frac{1}{Re Da} U \quad (12)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \frac{\mu_{nf}}{\rho_{nf} \cdot \gamma_f} \frac{1}{(1-\varphi)^{2.5}} \nabla^2 V - \frac{(\rho\beta)_{nf}}{\rho_{nf} \beta_f} \frac{1}{Re Da} V + \frac{(\rho\beta)_{nf}}{\rho_{nf} \beta_f} Ri \theta - \frac{Ha^2}{Re} V \quad (13)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \frac{1}{Re Pr} \nabla^2 \theta \quad (14)$$

And the dimensionless governing parameters:

$$Pr = \frac{\nu_f}{\alpha_f}, Da = \frac{K}{L^2}, Re = \frac{U_0 L}{\nu_f}, Gr = \frac{g \beta_f (T_h - T_c) L^3}{\nu_f^2}, Ri = \frac{Gr}{Re^2}, Ha = B_0 L \sqrt{\frac{\sigma_{nf}}{\rho_{nf} \cdot \gamma_f}}$$

where Pr , Da , Re , Gr , Ri and Ha are Prandtl number, Darcy number, Reynolds number, Grashof number, Richardson number and Hartmann number, respectively.

3.3. Boundary Conditions (dimensionless)

The dimensionless boundary conditions for the solving the governing (11)–(14) are as follows (Table III):

4. NUMERICAL TECHNIQUE

The Galerkin Weighted residual finite-element method is used to convert the governing partial differential equations into a set of integral equations, each equation term is integrated using the Gauss quadrature method. After applying boundary conditions, the resulting nonlinear algebraic equations are modified and transformed into linear algebraic equations using Newton's method. Finally, the Triangular factorization method is utilized to solve these linear equations.

According to Table IV, the average Nusselt number was determined for various grids. The table clearly demonstrates that the grid with 6743 nodes and 12270 elements yields a desirable solution for the numerical study at hand.

4.1. Program Validation

Fig. 2 displays a comparison of streamlines and isotherms in the graphical solution. The figures indicate a remarkable consistency between the two results, highlighting a strong agreement between them.

TABLE III: DIMENSIONLESS BOUNDARY CONDITIONS [15]

Walls	Dimensionless boundary conditions
Top wall	$U = 1, V = 0, \theta = 0$
Left wall	$U = 0, V = 0, \partial\theta/\partial X = 0$
Right wall	$U = 0, V = 0, \partial\theta/\partial X = 0$
Bottom wall	$U = -1, V = 0, \theta = 0$
Constant heated obstacle	$U = 0, V = 0, \theta = 1$

TABLE IV: GRID SENSITIVITY CHECK AT $Pr = 0.71$, $Ri = 1$ AND $Ha = 50$

Nodes	832	1114	1725	6743	24136
Elements	1601	2146	3328	12270	48468
Nu_{av}	8.64276	8.66746	8.67727	8.70372	8.70841
Time (s)	8	11	14	20	35

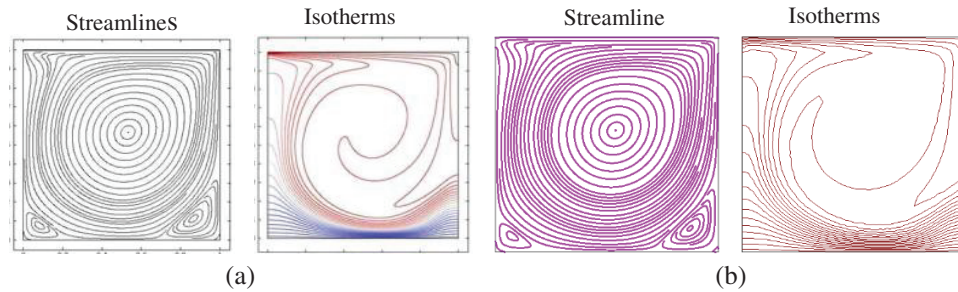


Fig. 2. Comparison between isotherms and streamlines for numerical solution of (a) Shaha et al. [17] $Ra = 10^5$, $Re = 100$, $Ha = 50$ and (b) present study.

5. RESULTS AND DISCUSSION

In this section, the numerical and graphical results of simulating the flow of nanofluid in a rhombus-shaped enclosure filled with porous media containing a heated obstacle are presented. The results have been obtained for the Darcy number ranging from $Da = 1e^{-5}$, $1e^{-4}$, $1e^{-3}$, $1e^{-2}$ and the Richardson number $Ri = 0.1$, 1 , 5 , 10 .

5.1. Variation of Darcy Number

Fig. 3 illustrates the variation of streamlines at different Darcy numbers. The figures clearly demonstrate that when symmetric boundary conditions are applied to the top and bottom sides of the enclosure, the flow exhibits symmetry along this line. As the Darcy number and buoyancy force increase, the vortices' centers move in the upward and downward directions towards the lid, indicating a stronger influence of the moving lid.

At $Ri = 1$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$, the isotherms in Fig. 4 indicate that conduction dominates the heat transfer. As the Darcy number increases, the isotherms become condensed near the upper and lower side walls, indicating an increase in heat transfer through convection. The presence of a thermal boundary layer can be observed from the isotherms at $Da = 1e^{-4}$. When $Da = 1e^{-3}$, the isotherms show pure conduction. A similar temperature distribution is seen for $Da = 1e^{-2}$.

Fig. 5 depicts the variation of the local Nusselt number along the X-axis of the enclosure for different Darcy numbers, with $Ri = 1$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$. The figure shows distinct curves representing the minimum and maximum values. It can be observed that at $Da = 1e^{-3}$, there is a substantial change in the local Nusselt number. However, as the Darcy number increases, the impact on the Nusselt number becomes less significant. Fig. 6 illustrates the changes in the velocity profile along the X-axis of the enclosure for various Darcy numbers, considering $Ri = 1$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$. The figure displays distinct curves representing the maximum and minimum values. Specifically, for X values less than 0.6, the velocity curves exhibit higher values, while for X values greater than 0.6, the velocity curves show lower values. In Fig. 7, the dimensionless temperature is plotted along the X-axis for various Darcy numbers, while considering $Ri = 1$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$. The figure indicates that as the Darcy numbers increase, the temperature values also increase. The parabolic shape of the temperature profile is a result of the symmetric shape of the heated

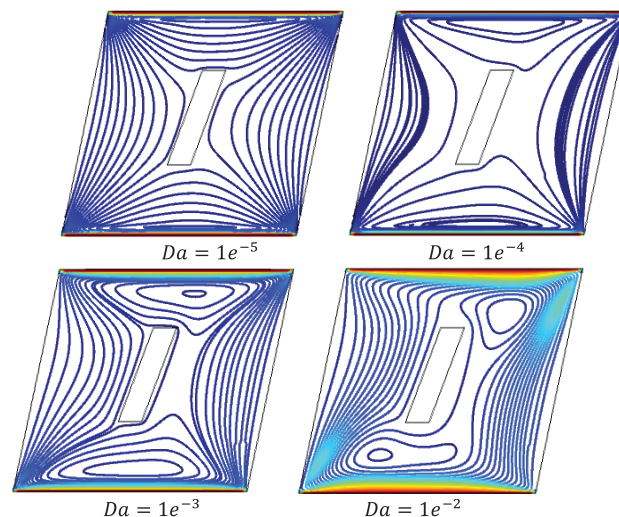


Fig. 3. Streamlines at different values of Darcy numbers $Da = 1e^{-5} - 1e^{-2}$ when $Ri = 1$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$.

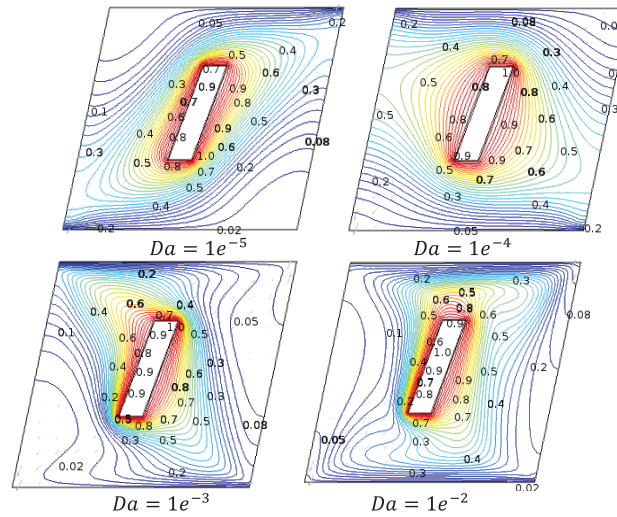


Fig. 4. Isotherms at different values of Darcy numbers $Da = 1e^{-5} - 1e^{-2}$ when $Ri = 1$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$.

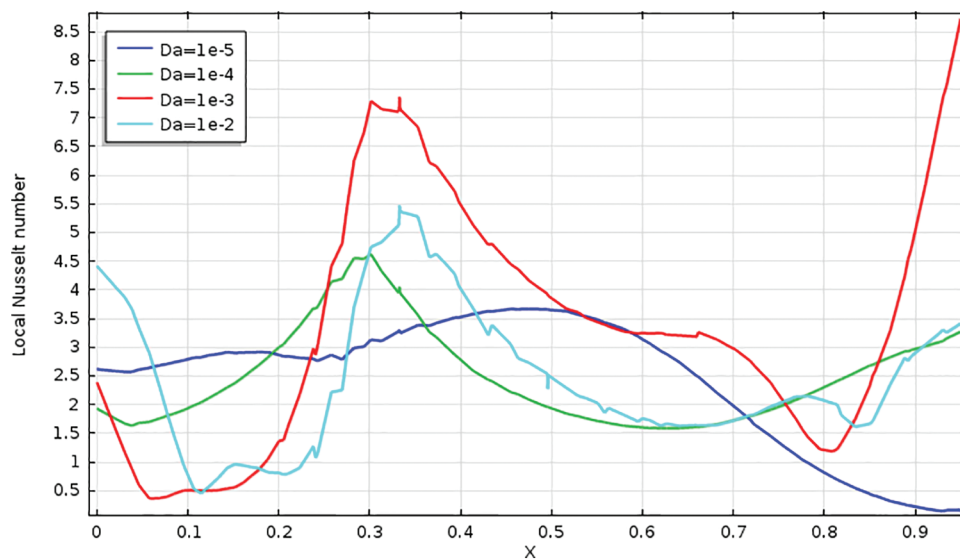


Fig. 5. Variation of local Nusselt number along X – axis of the enclosure with different values of Darcy numbers $Da = 1e^{-5} - 1e^{-2}$ when $Ri = 1$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$.

block. Lower Darcy numbers exhibit a less significant change in temperature, whereas higher Darcy numbers demonstrates a more noticeable effect on temperature.

5.2. Heat Transfer Rates

This section presents a numerical simulation of nanofluid flow within a rhombus-shaped enclosure filled with porous media, where a constant heated obstacle is present. The relationship between the average Nusselt number and Darcy numbers, considering various Richardson numbers for heat transfer rates, is depicted in Fig. 8. It can be observed that the average Nusselt number increases as the Darcy number increases. Generally, higher Darcy number result in higher temperature gradients, which consequently lead to an increase in the average Nusselt number across different Richardson numbers.

Fig. 9 displays the relationship between the average fluid temperature and the Darcy number within the enclosure, considering different values of the Richardson number ranging from 0.1 to 10, while keeping the Hartmann number at 50 and the porosity at 5%. The findings indicate that an elevated Richardson number within the porous enclosure leads to an augmentation in the average fluid temperature. Furthermore, the rate at which heat transfer increases distinctly relies on the value of the Darcy number.

5.3. Variation of Richardson Number

This section presents a numerical simulation of the flow of nanofluid in a rhombus-shaped enclosure filled with porous media, where there is a constant heated obstacle. The figures, Figs. 10 and 11, demonstrate the changes in streamlines and isotherms within the constant heated obstacle for various values of the Richardson number. The parameters used in the simulation are $Da = 1e^{-5}$, $Pr =$

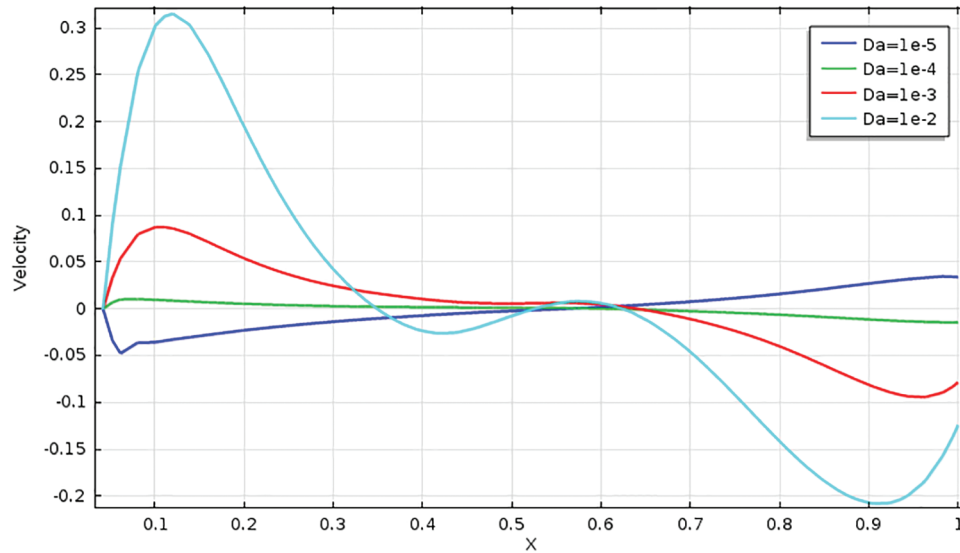


Fig. 6. Variation of velocity profiles along X – axis of the enclosure with different values of Darcy numbers $Da = 1e^{-5} - 1e^{-2}$ when $Ri = 1$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$.

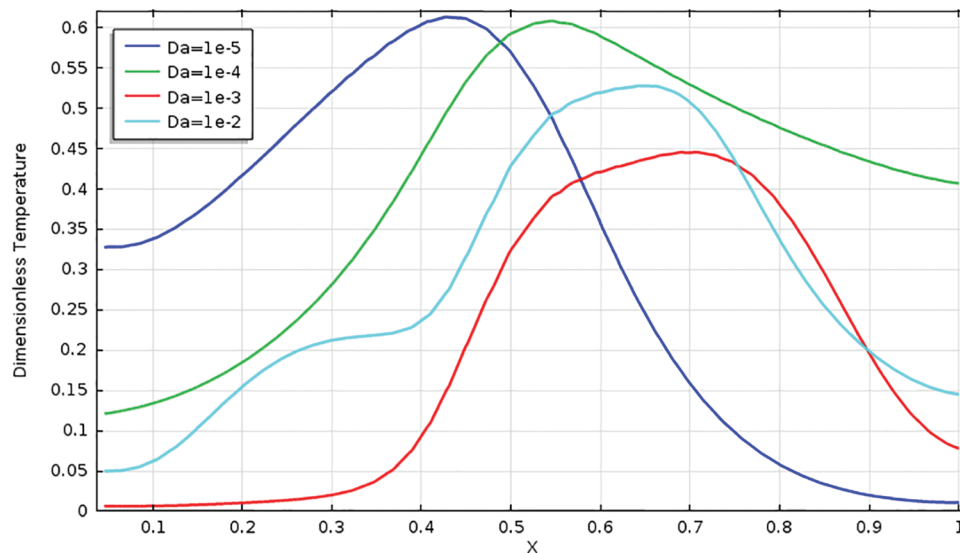


Fig. 7. Variation of dimensionless temperature along X – axis of the enclosure with different values of Darcy numbers $Da = 1e^{-5} - 1e^{-2}$ when $Ri = 1$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$.

6.2, $Ha = 50$ and $\phi = 5\%$. It is evident from the figures that symmetrical boundary conductions exist in the system. At lower values of the Richardson number, the fluid inside the enclosure raises more, and the motion of the lid becomes more vigorous. Conversely, for higher Richardson numbers, three elliptic-shaped regions, referred to as ‘eyes’ form inside the enclosure, indicating increased flow strength. The isotherms shown in Fig. 11 indicate that heat transfer is primarily dominated by conduction. As the Richardson number increases, the isotherms tend to concentrate near the constant heated obstacle, and their lines become more curved, suggesting enhanced heat transfer through convection.

Fig. 12 illustrates the local Nusselt number along the horizontal wall of the enclosure for various Richardson numbers, considering parameters $Da = 1e^{-5}$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$. The curves in the plot exhibit minimum and maximum shapes. For values of X less than 0.3, the curves take on a minimum shape, while for values of X greater than 0.3, the curves assume a maximum shape. Additionally, the absolute value of the local Nusselt number, representing the heat transfer rate, increases as the Richardson number rises. In Fig. 13, the velocity profiles along the horizontal wall of the enclosure are depicted for different Richardson numbers, taking into account parameters $Da = 1e^{-5}$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$. It is evident from the figure that both the maximum and minimum velocities increase as the Richardson number increases. The change in velocity is relatively small for lower Richardson numbers, but it becomes more pronounced for higher Richardson numbers. Fig. 14 illustrates the temperature profiles along the horizontal wall of the enclosure for various Richardson numbers, considering parameters $Da = 1e^{-5}$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$. It can be observed from the figure that the temperature decreases as the Richardson number increases.

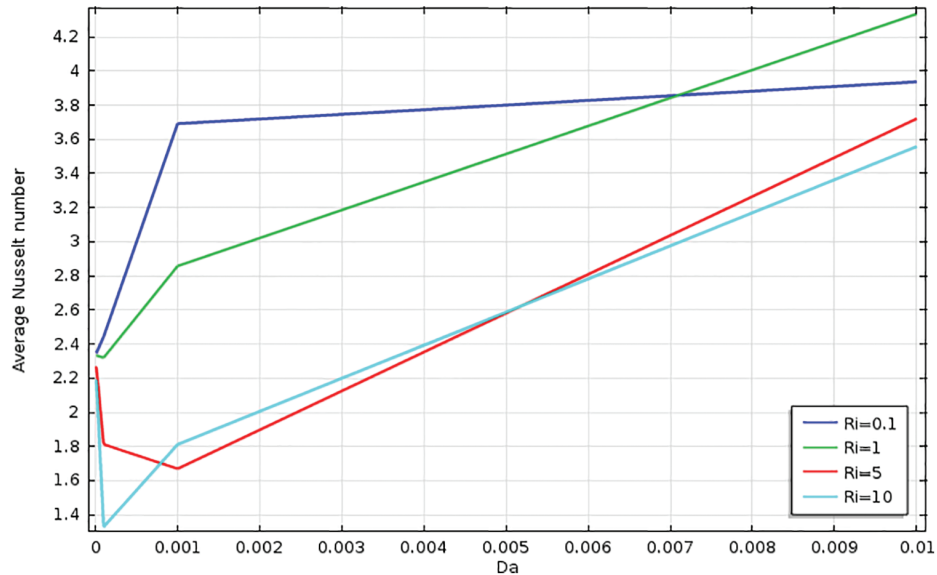


Fig. 8. Variation of the average Nusselt number versus Darcy number $Da = 1e^{-5} - 1e^{-2}$ with different values of Richardson number $Ri = 0.1$ to 10 when $Ha = 50$ and $\varphi = 5\%$.

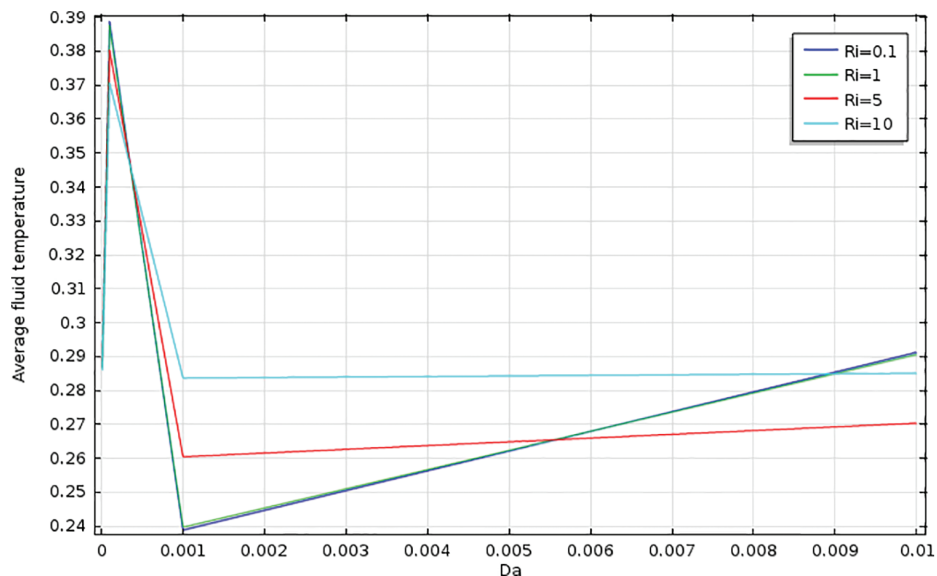


Fig. 9. Variation of the average fluid temperature versus Darcy number $Da = 1e^{-5} - 1e^{-2}$ with different values of solid volume fraction of nanoparticles $\varphi = 0 - 0.1$ when $Ri = 1$, $Ha = 50$.

The change in temperature is relatively less significantly for lower Richardson numbers, whereas for higher Richardson numbers, the temperature exhibits a more pronounced and significant change.

5.4. Heat Transfer Rates

This section presents a numerical simulation of nanofluid flow in a rhombus-shaped enclosure filled with porous media, featuring a constant heated obstacle. The simulation focuses on the average Nusselt number in relation to the Richardson number for different values of the Darcy number, while considering parameters $Pr = 6.2$, $Ha = 50$ and $\varphi = 5\%$.

Fig. 15 displays the relationship between the average Nusselt number and the Richardson number for various Darcy numbers. It can be observed from the figure that as the Darcy number increases, the average Nusselt number also increases. This can be attributed to higher temperature gradients associated with larger Darcy numbers. Overall, the average Nusselt number exhibits an increasing trend with Darcy numbers for different values of the Richardson number.

Fig. 16 presents the relationship between the average fluid temperature and the Richardson number inside the constant heated enclosure, considering different Darcy numbers, while keeping $\varphi = 5\%$, $Ha = 50$, $Pr = 6.2$ constant. It is evident from the figure that increasing the Darcy number inside the constant heated porous enclosure leads to an increase in the average fluid temperature. Additionally, the rate of increment in heat transfer is dependent on the specific value of the Darcy number.

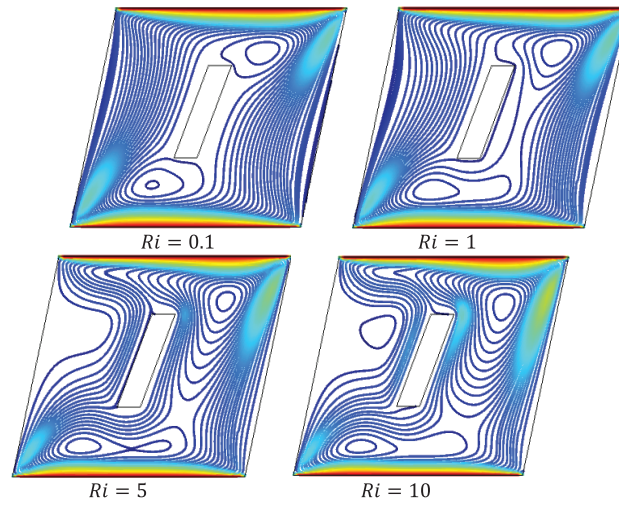


Fig. 10. Streamlines at different values of Richardson numbers $Ri = 0.1, 1, 5, 10$ when $Da = 1e^{-5}$, $Pr = 6.2$, $Ha = 50$ and $\phi = 5\%$.

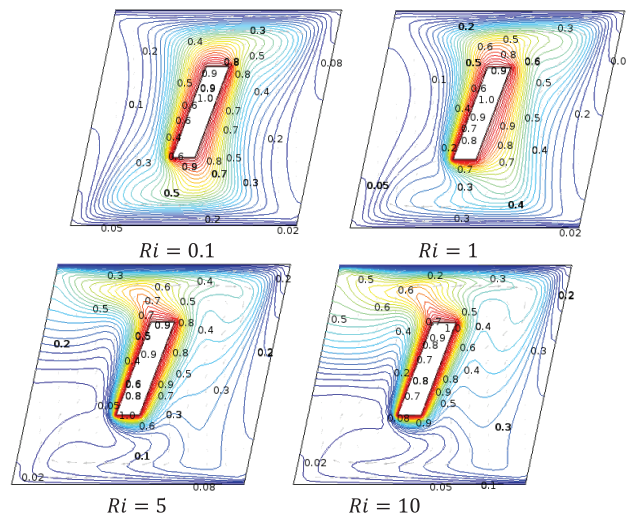


Fig. 11. Isotherms at different values of Richardson numbers $Ri = 0.1, 1, 5, 10$ when $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$.

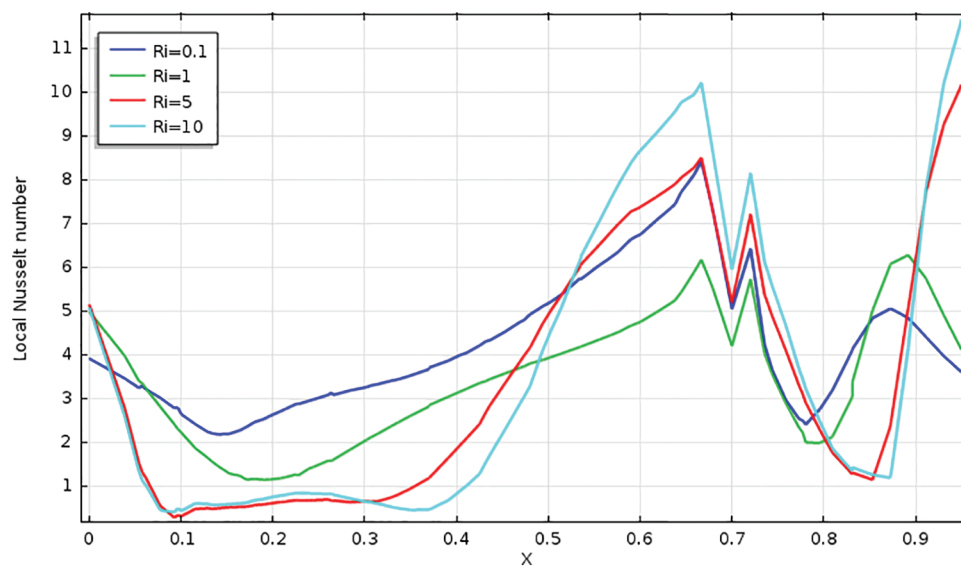


Fig. 12. Variation of local Nusselt number along X – axis of the enclosure with different values of Richardson numbers $Ri = 0.1, 1, 5, 10$ when $Da = 1e^{-5}$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$.

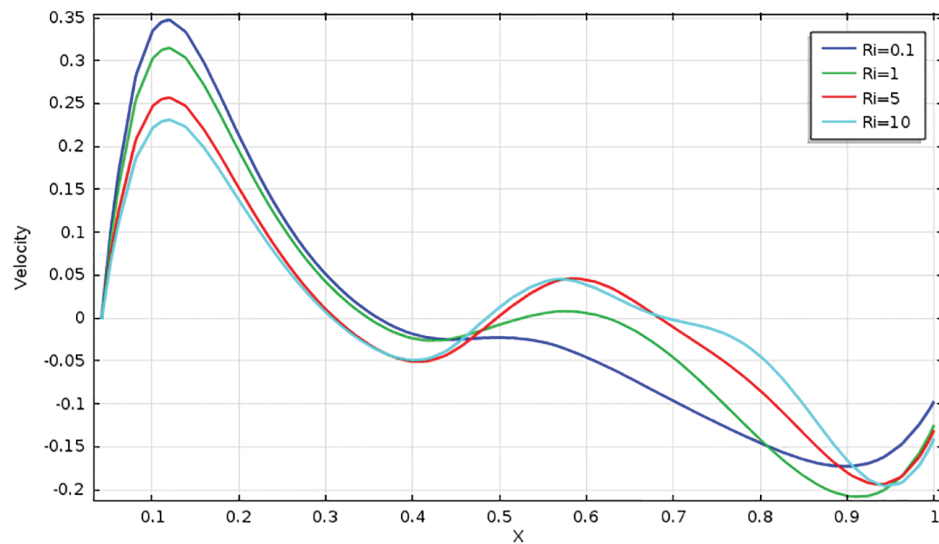


Fig. 13. Variation of velocity profiles along X – axis of the enclosure with different values of Richardson numbers $Ri = 0.1, 1, 5, 10$ when $Da = 1e^{-5}$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$.

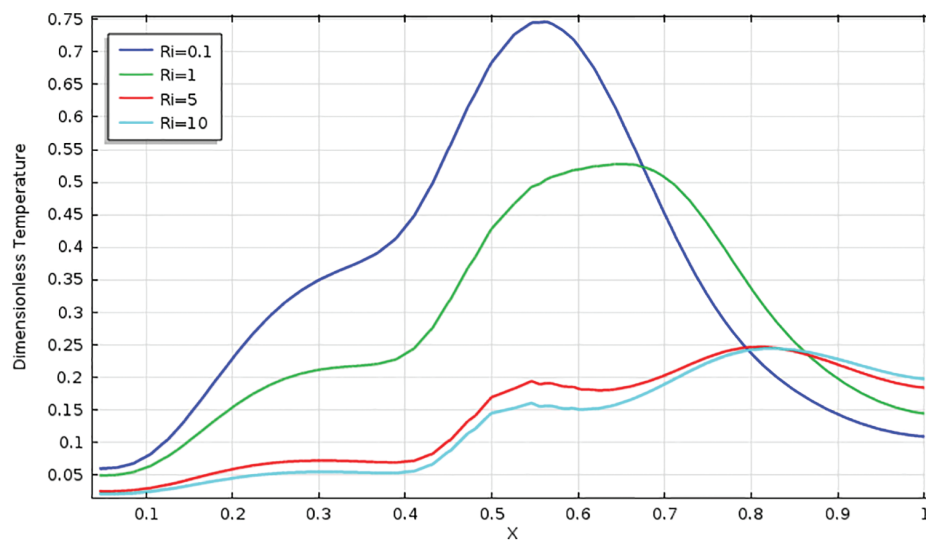


Fig. 14. Variation of dimensionless temperature along X – axis of the enclosure with different values of Richardson numbers $Ri = 0.1, 1, 5, 10$ when $Da = 1e^{-5}$, $Ha = 50$, $Pr = 6.2$ and $\phi = 5\%$.

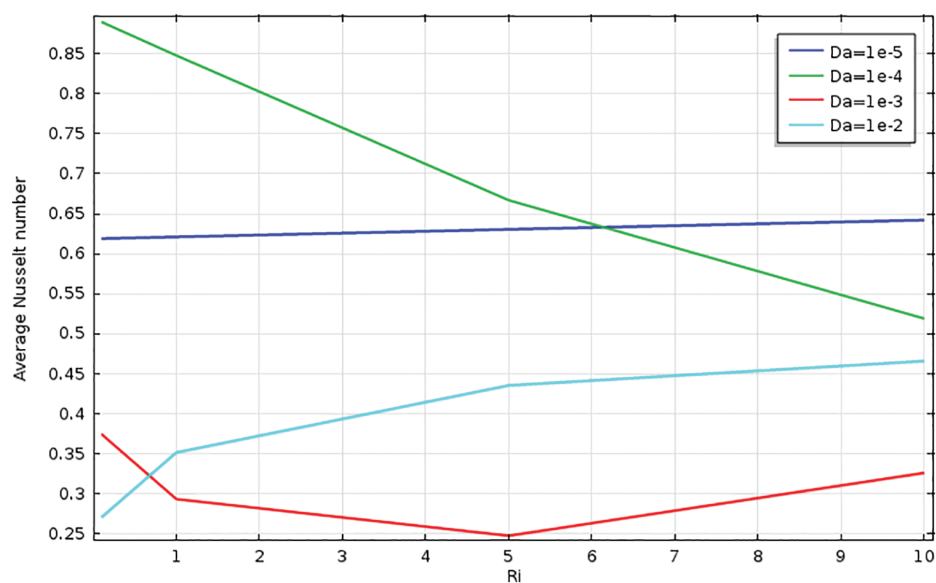


Fig. 15. Variation of the average Nusselt number versus Richardson numbers $Ri = 0.1, 1, 5, 10$ with different values of Darcy number while $\phi = 5\%$, $Ha = 50$, $Pr = 6.2$.

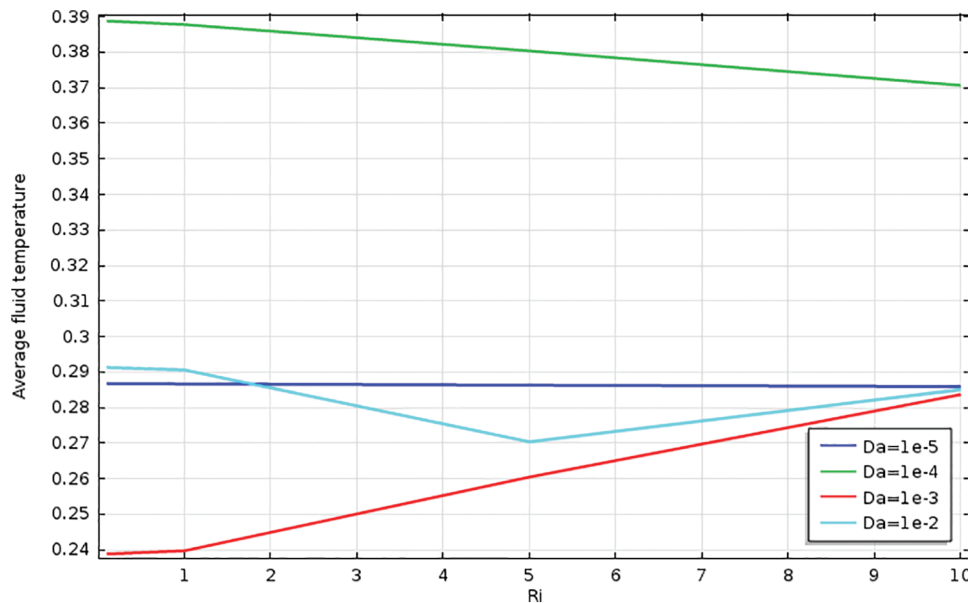


Fig. 16. Variation of the average fluid temperature versus Richardson numbers $Ri = 0.1, 1, 5, 10$ with different values of Darcy number while $\phi = 5\%$, $Ha = 50$, $Pr = 6.2$.

6. CONCLUSION

The impact of nanofluid flow in a rhombus-shaped porous on a constant heated obstacle has been conducted. This study encompasses a wide range of dimensionless groups, including the Darcy number and Richardson number. Both the Darcy number and the movement of the lids have a substantial impact on the flow and thermal fields within the enclosure. The Richardson number serves as a useful control parameter for heat transfer in fluid flow through porous media enclosures. The velocity of the fluid is enhanced as the solid volume fraction of nanoparticles increases, resulting in an increase in the heat transfer rate. The highest rate of heat transfer is observed in the vicinity of the heated obstacle within the enclosure. Furthermore, an increase in the Darcy number of the heated enclosure leads to an augmentation in the heat transfer rate.

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