A Direct Proof for Fermat's Last Theorem using Ramanujan-Nagell Equation

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Abstract — Fermat's Last Theorem states that it is impossible to find positive integers A, B and C satisfying the equation:

\[ A^n + B^n = C^n \]

where \( n \) is any integer > 2.

Taking the proofs of Fermat for the index \( n = 4 \), and Euler for \( n = 3 \), it is sufficient to prove the theorem for \( n = p \), any prime > 3 [1].

We hypothesize that all \( r, s \) and \( t \) are non-zero integers in the equation:

\[ r^p + s^p = t^p \]

and establish contradiction.

Just for supporting the proof in the above equation, we have another equation:

\[ x^3 + y^3 = z^3 \]

Without loss of generality, we assert that both \( x \) and \( y \) as non-zero integers; \( z^3 \) a non-zero integer; \( z \) and \( z^2 \) irrational.

We create transformed equations to the above two equations through parameters, into which we have incorporated a Ramanujan-Nagell equation. Solving the transformed equations, we prove the theorem.

Keywords — Transformed Fermat's Equations through Parameters.

I. INTRODUCTION

Around 1637, Pierre-de-Fermat, the French mathematician wrote in the margin of a book that the equation \( A^n + B^n = C^n \) has no solution in integers \( A, B \) and \( C \), if \( n \) is any integer >2. Fermat stated in the margin of the book that he himself had found a marvelous proof of the theorem, but the margin was too narrow to contain it. His proof is available only for the index \( n=4 \), using infinite descent method.

Many mathematicians like Sophie Germain, E.E. Kummer had proved the theorem for particular cases. Number theory has been developed leaps and bounds by the immense contributions by a lot of mathematicians. Finally, after 350 years, the theorem was completely proved by Prof. Andrew Wiles, using highly complicated mathematical tools and advanced number theory [2], [3]. Here we are trying an elementary proof.

II. ASSUMPTIONS

1) We initially hypothesize that all \( r, s \) and \( t \) are non-zero integers satisfying the equation:

\[ r^p + s^p = t^p \]

where \( p \) is any prime > 3, with \( \gcd(r, s, t) = 1 \) and establish a contradiction in this proof.

2) Just for supporting the proof in the above equation, we have taken another equation:

\[ x^3 + y^3 = z^3; \gcd(x, y, z^2) = 1 \]

Without loss of generality, we can have both \( x \) and \( y \) as non-zero integers, \( z^3 \) a non-zero integer; both \( z \) and \( z^2 \) irrational. Since we prove the theorem only in the equation \( r^p + s^p = t^p \) for all possible integral values of \( r, s \) and \( t \) we have the choice in having \( x=41; y=36; z^3=41^3+36^3=11 \times 19 \times 79 \times 7 \).
3) By trial and error method, we have created the transformation equations to \( x^3 + y^3 = z^3 \) and \( r^p + s^p = t^p \) using parameters called \( a, b, c, d, e \) and \( f \). Creation of such transformation equations could be done in thousands of ways, but giving a proof is most difficult and rare. Every time the rational terms in (8) we derive from the transformed equations got cancelled out on both sides. After enormous random trials, the formulation of transformed equations was achieved to bring out the results for proving the theorem.

4) Into the transformed equations we have incorporated the Ramanaujan-Nagell equation \( 2n^2 = 7 + \ell^2 \), which has just five solutions given by:

\[
(n, \ell) = \{(3,1); (4,3); (5,5); (7,11); (15,181)\}
\]

In this proof, we take only the solutions:

\[2^7 = 7 + 11^2\]

where \(\ell = 11\). Hence \( z^3 = 7 \times 11 \times 19 \times 79 = 7 \times 19 \times 79 \times \ell \).

**Proof.** By random trials, we have created the following equations,

\[
\left(a\sqrt{41z^7} + b\sqrt{2n^2} \right)^2 + \left(c\sqrt{79 + d\sqrt{\ell^2}} \right)^2 = \left(e\sqrt{7} + f\sqrt{19} \right)^2
\]

and

\[
\left(a\sqrt{19} - b\sqrt{st} \right)^2 + \left(c\sqrt{41 - d\sqrt{r}} \right)^2 = \left(e\sqrt{7/3} - f\sqrt{41z^7} \right)^2
\]

as the transformed equations of \( x^3 + y^3 = z^3 \) and \( r^p + s^p = t^p \) respectively through the parameters called \( a, b, c, d, e \) and \( f \).

Here we have taken \( x = 41; y = 36; \)

\[z^3 = x^3 + y^3 = 41^3 + 36^3 = 7 \times 11 \times 19 \times 79 = 7 \times 19 \times 79 \times \ell \).

where \(\ell = 11\).

We have incorporated the Ramanaujan-Nagell equation \( 2n^2 = 7k^2 + \ell^2 \) into the transformation equations. We may have:

\[a\sqrt{41z^7} + b\sqrt{2n^2} = \sqrt{x^3} \quad (2)\]

\[a\sqrt{19} - b\sqrt{st} = \sqrt{r^p} \quad (3)\]

\[c\sqrt{79 + d\sqrt{\ell^2}} = \sqrt{y^3 2^{5/2}} \quad (4)\]

\[c\sqrt{41 - d\sqrt{r}} = \sqrt{s^p 7^{5/3}} \quad (5)\]

\[e\sqrt{7} + f\sqrt{19} = \sqrt{z^7} \quad (6)\]

\[e\sqrt{7/3} - f\sqrt{41z^7} = \sqrt{t^p \ell^{7/3}} \quad (7)\]

Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get:

\[
\frac{a}{\sqrt{2n^2} r^p} = \left(\sqrt{x^3 st} + \sqrt{2n^2 r^p \ell} \right) \left(\sqrt{41stz^7} + \sqrt{19z^7} \right)
\]

\[
\frac{b}{\sqrt{2n^2} r^p} = \left(\sqrt{19x^3} - \sqrt{41z^7} \right) \left(\sqrt{41stz^7} + \sqrt{19z^7} \right)
\]
\[c = (\sqrt{2^{3\nu} y^r} + \sqrt{7^{7/3} \ell^{1/3} s^p}) / \left(\sqrt{7} r + \sqrt{41} \ell^{7/3}\right)\]
\[d = (41 \times 2^{3\nu} y^j - \sqrt{79} \times 7^{3/3} s^p) / \left(\sqrt{7} r + \sqrt{41} \ell^{7/3}\right)\]
\[e = (z^2 \sqrt{41} + \sqrt{19} \ell^{7/3} s^p) / \left(\sqrt{7} z + \sqrt{19} \times 7^{7/3}\right)\]
\[f = (\sqrt{7^{1/3} z^3} - \sqrt{7} \times \ell^{1/3} t^p) / \left(\sqrt{7} z + \sqrt{19} \times 7^{7/3}\right)\]

From (2) & (4), we get:
\[\sqrt{2^{3\nu}} \times \sqrt{2^{3\nu}} = \left(\sqrt{x^3} - a \sqrt{41} z^3\right) / \left(\sqrt{b} \sqrt{y^3}\right)\]
i.e.,
\[2^n = \left(c \sqrt{79} x^3 + (d) \sqrt{\ell^{7/3} x^3} - (ac) \sqrt{41} \times 79z^3 - (ad) \sqrt{41} \ell^{1/3} z^3\right) / \left(b \sqrt{y^3}\right)\]

From (5) & (7), we get:
\[\sqrt{7^{7/3}} \times \sqrt{7^{7/3}} = \left(c \sqrt{41} - d \sqrt{r}\right) / \left(\sqrt{\ell^{1/3} + f \sqrt{41} z}\right) \]
i.e.,
\[7 = \left\{c \sqrt{41} x^3 + (cf) \sqrt{\ell^{7/3} x^3} - (d) \sqrt{\ell^{7/3} x^3} - (df) \sqrt{41} \ell^{7/3} z\right\} / \left(e \sqrt{y^3}\right)\]

From (4) & (7), we get:
\[\sqrt{\ell^{7/3}} \times \sqrt{\ell^{7/3}} = \left(\sqrt{2^{3\nu}} y^r + c \sqrt{79} - f \sqrt{41} z\right) / \left(d \sqrt{t^p}\right)\]
i.e.,
\[\ell^2 = \left\{e \sqrt{2^{3\nu} \ell^{7/3} y^3} - (f) \sqrt{79} z + (ce) \sqrt{79} \times 79 + (cf) \sqrt{41} \times 79\right\} / \left(d \sqrt{r^p}\right)\]

Substituting the above equivalent values of 2^n; 7 and \(\ell^2\) in the Ramanujan-Nagell equation \(2^n = 7 + \ell^2\) after multiplying both sides by \(\left\{bde\right\}\), we get:
\[\left\{de\right\} \sqrt{y^3 \ell^{7/3}} \left\{c\right\} \sqrt{79} x^3 + (d) \sqrt{\ell^{7/3} x^3} - (ac) \sqrt{41} \times 79z^3 - (ad) \sqrt{41} \ell^{1/3} z^3\]
\[= \left\{bd\right\} \sqrt{y^3 \ell^{7/3}} \left\{c\right\} \sqrt{41} x^3 + (cf) \sqrt{41} z - (d) \sqrt{\ell^{7/3} x^3} - (df) \sqrt{41} \ell^{7/3} z\]
\[+ \left\{be\right\} \sqrt{y^3 \ell^{7/3}} \left\{c\right\} \sqrt{2^{3\nu} \ell^{7/3} y^3} - (f) \sqrt{79} z + (ce) \sqrt{79} \times 79\]
\[-(ce) \sqrt{79} \times 7^{7/3} + (cf) \sqrt{41} \times 79\]

We will work out all rational terms available in (8), after multiplying both sides by:
\[\left(41 \times 79z^3 + \sqrt{19} \times 2^n z^2\right) \sqrt{79r} + \sqrt{41} \ell^{7/3} \left(\sqrt{7} \times 41z + \sqrt{19} \times 7^{7/3}\right)^2\]
to be free from denominators on the parameters and again multiplying by:
\[\left\{\sqrt{7} \times 19 \times 41 \times \ell^{7/3} t^p\right\}\]

for getting some rational terms.
I term in LHS of Equation (8), after multiplying by the respective terms, and substituting for \{(cd)e\}
There is no rational part in this term.

II term in LHS of equation (8), after multiplying by the respective terms, and substituting for \{d^2e\} is:

\[
= \sqrt{s^2 t^p} \left( \sqrt{79 x^3 + 19 z^3} \right) \left( \sqrt{7 \times 41 z + 19 \times 7^{1/3}} \right) \left( \sqrt{7 \times 19 \times 41 \times z^2 t^{4/3}} \right) \\
\times \left( 2^{2^{0.5} y^3 + 7^{0.5} z^{5.5} s^p} \right) \left( 41 \times 2^{2^{0.5} y^3} - 19 \times 7^{3/5} x^3 \right) \left( z^2 \sqrt{41 + 19 t^2} \right) 
\]

There is no rational part in this term.

III term in LHS of equation (8), after multiplying by the respective terms, and substituting for \{a(cde)\} is:

\[
= \left( -\sqrt{s^2 t^p} \right) \left( 41 \times 79 z^2 \right) \left( 7 \times 41 z + 19 \times 7^{1/3} \right) \left( \sqrt{7 \times 19 \times 41 \times z^4 t^{2/3}} \right) \left( \sqrt{7^{2/3} t^{1/3} x} \right) \\
\times \left( 2^{2^{0.5} y^3 + 7^{0.5} z^{5.5} s^p} \right) \left( 41 \times 2^{2^{0.5} y^3} - 19 \times 7^{3/5} x^3 \right) \left( z^2 \sqrt{41 + 19 t^2} \right) 
\]

On multiplying by,

\[
\left\{ \left( -\sqrt{s^2 t^p} \right) \left( 41 \times 79 z^2 \right) \left( 7 \times 41 z + 19 \times 7^{1/3} \right) \left( \sqrt{7^{2/3} t^{1/3} x} \right) \left( \sqrt{7 \times 19 \times 41 \times z^4 t^{2/3}} \right) \left( \sqrt{7^{2/3} t^{1/3} x} \right) \left( 41 \times 2^{2^{0.5} y^3} - 19 \times 7^{3/5} x^3 \right) \left( z^2 \sqrt{41 + 19 t^2} \right) \right\} 
\]

we get:

\[
\left\{ \left( -\sqrt{s^2 t^p} \right) \left( 41 \times 79 z^2 \right) \left( 7 \times 41 z + 19 \times 7^{1/3} \right) \left( \sqrt{7^{2/3} t^{1/3} x} \right) \left( \sqrt{7 \times 19 \times 41 \times z^4 t^{2/3}} \right) \left( \sqrt{7^{2/3} t^{1/3} x} \right) \left( 41 \times 2^{2^{0.5} y^3} - 19 \times 7^{3/5} x^3 \right) \left( z^2 \sqrt{41 + 19 t^2} \right) \right\} 
\]

where \( y = 36 \); this term will be discussed later on.

IV term in LHS of equation (8), after multiplying by the respective terms, and substituting for \{a(de)\} is:

\[
= \left( -\sqrt{s^2 t^p} \right) \left( 41 \times 79 z^2 \right) \left( 7 \times 41 z + 19 \times 7^{1/3} \right) \left( \sqrt{7^{2/3} t^{1/3} x} \right) \left( \sqrt{7 \times 19 \times 41 \times z^4 t^{2/3}} \right) \left( \sqrt{7^{2/3} t^{1/3} x} \right) \\
\times \left( 2^{2^{0.5} y^3 + 7^{0.5} z^{5.5} s^p} \right) \left( 41 \times 2^{2^{0.5} y^3} - 19 \times 7^{3/5} x^3 \right) \left( z^2 \sqrt{41 + 19 t^2} \right) 
\]

On multiplying by,

\[
\left\{ \left( -\sqrt{s^2 t^p} \right) \left( 41 \times 79 z^2 \right) \left( 7 \times 41 z + 19 \times 7^{1/3} \right) \left( \sqrt{7^{2/3} t^{1/3} x} \right) \left( \sqrt{7 \times 19 \times 41 \times z^4 t^{2/3}} \right) \left( \sqrt{7^{2/3} t^{1/3} x} \right) \left( 41 \times 2^{2^{0.5} y^3} - 19 \times 7^{3/5} x^3 \right) \left( z^2 \sqrt{41 + 19 t^2} \right) \right\} 
\]

we get

\[
\left\{ \left( 2^{2^{0.5} y^3 + 7^{0.5} z^{5.5} s^p} \right) \left( 41 \times 2^{2^{0.5} y^3} - 19 \times 7^{3/5} x^3 \right) \left( z^2 \sqrt{41 + 19 t^2} \right) \right\} 
\]

This term will be discussed later on.

I term in RHS of equation (8), after multiplying by the respective terms and substituting for \{b(cde)\} is:

\[
= \left( t^p \sqrt{41 y^3 t^{1/3}} \right) \left( \sqrt{7 \times 41 z + 19 \times 7^{1/3}} \right) \left( \sqrt{7 \times 19 \times 41 \times z^4 t^{2/3}} \right) \left( \sqrt{19x^3 - 41 \times t^2 z^3} \right) \\
\times \left( 2^{2^{0.5} y^3 + 7^{0.5} z^{5.5} s^p} \right) \left( 41 \times 2^{2^{0.5} y^3} - 19 \times 7^{3/5} x^3 \right) \left( z^2 \sqrt{41 + 19 t^2} \right) 
\]

There is no rational part in this term.

II term in RHS of (8), after multiplying by the respective terms and substituting for \{b(cde)\} is:

\[
= \left( 41 \times \sqrt{y^3 t^p} \right) \left( \sqrt{7 \times 41 z + 19 \times 7^{1/3}} \right) \left( \sqrt{7 \times 19 \times 41 \times z^4 t^{2/3}} \right) \left( \sqrt{19x^3 - 41 \times t^2 z^3} \right) 
\]
\[ x\left(\sqrt{2^{3n/2} y^r + \sqrt[5/3]{\ell^{6/3} s^r}}\right)\left(\sqrt[41]{2^{3n/2} y^r} - \sqrt[79]{7^{1/3} s^r}\right)\left(\sqrt[7]{7^{1/3} z^r} - \sqrt[7]{\ell r^{6/3} s^r}\right) \]

(i) Rational part in this term

\[ = \left\{41 \times \sqrt[3]{y^r z^r}\right\} \left[\sqrt[79]{7^{1/3} s^r}\left(\sqrt[7]{7^{1/3} z^r} - \sqrt[7]{\ell r^{6/3} s^r}\right)\right] \]

\[ = \left[-\left(7^3 \times 19 \times 41\right)\left(s^r \sqrt[6]{r^{3/1}}\right)\left(z^r\right)\right] \left(41 \times y^r z^r\right) \left(\sqrt[79]{7^{1/3} s^r}\right) \left(\sqrt[7]{7^{1/3} z^r} - \sqrt[7]{\ell r^{6/3} s^r}\right) \]

(ii) Also on multiplying by:

\[ \left\{41 \times \sqrt[3]{y^r z^r}\right\} \left[\sqrt[79]{7^{1/3} s^r}\left(\sqrt[7]{7^{1/3} z^r} - \sqrt[7]{\ell r^{6/3} s^r}\right)\right] \]

we get

\[ \left\{7^3 \times 19 \times 41^2\right\} \left(\ell r^{6/3} s^r\right) \left(\sqrt[79]{7^{1/3} s^r}\right) \left(\sqrt[7]{7^{1/3} z^r} - \sqrt[7]{\ell r^{6/3} s^r}\right) \]

Where \( y = 6^2 \); This term will be discussed later on.

III term in RHS of (8), after multiplying by the respective terms and substituting for \{bd\^2\} is:

\[ = -\left(\ell r^{3/1}\right) \left(\ell r^{6/3} s^r\right) \left(\sqrt[7]{7^{1/3} z^r} - \sqrt[7]{\ell r^{6/3} s^r}\right) \]

\[ \times \left(41 \times y^r z^r + \left(79 \times 7^{1/3} s^r\right) - 2 \left(41 \times 2^{3n/2} y^r\right) \sqrt[79]{7^{1/3} s^r}\right) \]

There is no rational part in this term.

IV term in RHS of (8), after multiplying by the respective terms and substituting for \{bd\ell f\} is:

\[ = -\left(\ell r^{6/3} s^r\right) \left(\sqrt[7]{7^{1/3} z^r} - \sqrt[7]{\ell r^{6/3} s^r}\right) \]

\[ \times \left(7^{1/3} z^r - \sqrt[7]{\ell r^{6/3} s^r}\right) \left(41 \times y^r z^r - \left(2 \times 2^{3n/2} y^r\right) \sqrt[79]{7^{1/3} s^r}\right) \]

There is no rational part in this term.

V term in RHS of (8), after multiplying by the respective terms and substituting for \{be\ell^2\} is:

\[ = \left(y^3 \sqrt[2^{3n/2} \times 7^{1/3} s^r}\right) \left(\sqrt[79]{7^{1/3} s^r} + \sqrt[41]{\ell^{6/3}}\right) \]

\[ \times \left(\sqrt[7]{7^{1/3} z^r} - \sqrt[7]{\ell r^{6/3} s^r}\right) \left(\ell^2 \sqrt[41]{7^{1/3} s^r} \times \sqrt[79]{7^{1/3} s^r}\right) \]

There is no rational part in this term.

VI term in RHS of (8), after multiplying by the respective terms and substituting for \{be\ell f\} is:

\[ = \left(y^3 \sqrt[2^{3n/2} \times 7^{1/3} s^r}\right) \left(\sqrt[79]{7^{1/3} s^r} + \sqrt[41]{\ell^{6/3}}\right) \]

\[ \times \left(\sqrt[7]{7^{1/3} z^r} - \sqrt[7]{\ell r^{6/3} s^r}\right) \left(\ell^2 \sqrt[41]{7^{1/3} s^r} \times \sqrt[79]{7^{1/3} s^r}\right) \]

There is no rational part in this term.

VII term in RHS of (8), after multiplying by the respective terms and substituting for \{bee\ell^2\} is:

\[ = \left(-y^3 \sqrt[2^{3n/2} \times 7^{1/3} s^r}\right) \left(\sqrt[79]{7^{1/3} s^r} + \sqrt[41]{\ell^{6/3}}\right) \]

\[ \times \left(\sqrt[7]{7^{1/3} z^r} - \sqrt[7]{\ell r^{6/3} s^r}\right) \left(2 \times 2^{3n/2} y^r\right) \sqrt[79]{7^{1/3} s^r} \sqrt[7]{\ell r^{6/3} s^r} \]

On multiplying by:

\[ \left(-y^3 \sqrt[2^{3n/2} \times 7^{1/3} s^r}\right) \left(\sqrt[79]{7^{1/3} s^r} + \sqrt[41]{\ell^{6/3}}\right) \]

\[ \times \left(\sqrt[7]{7^{1/3} z^r} - \sqrt[7]{\ell r^{6/3} s^r}\right) \left(2 \times 2^{3n/2} y^r\right) \sqrt[79]{7^{1/3} s^r} \sqrt[7]{\ell r^{6/3} s^r} \]

we get

\[ \left\{2 \times 7^3 \times 19 \times 41^2\right\} \left(\ell^2 s^r \sqrt[6]{r^{3/1}}\right) \left(z^r\right) \left(\ell^2 \sqrt[41]{7^{1/3} s^r} \times \sqrt[79]{7^{1/3} s^r}\right) \]
This term will be discussed later on.

VIII term in RHS of (8), after multiplying by the respective terms and substituting for \( \{bc(ef)\} \) is:

\[
\left(\sqrt{41 \times 79} y^z z^r \right) \left(\sqrt{79} r + \sqrt{41} \ell^{r-1} \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right)
\]

\[
\times \left( \ell^2 \sqrt{41} + \sqrt{19} \times 19 \ell^{r-1} \right) \left( \sqrt{2} \times 2 \times 19 \ell^r \right) \left( \sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right)
\]

(i) Rational part in this term

\[
\left(\left(\sqrt{41 \times 79} y^z z^r \right) \left(41 \ell^{r-1} \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right)
\]

\[
= \left(7 \times 19 \times 41 \ell^r \left(\sqrt{7} \times 19 \times 79 \ell^z \right)\right)
\]

Since \( x = 41; y = 62 \); and \( z^3 = 7 \times 19 \times 79 \), where \( \ell = 11 \).

(ii) Also, on multiplying by, we get

\[
\left(\left(\sqrt{41 \times 79} y^z z^r \right) \left(41 \ell^{r-1} \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right)
\]

we get

\[
\left(7 \times 19 \times 41 \ell^r \left(\sqrt{7} \times 19 \times 79 \ell^z \right)\right)\left(\ell^s \ell^t \right) = \left(7 \times 19 \times 41 \ell^r \right)
\]

where \( y = 6^2 \). This term will be discussed later on.

Case (1):

The rational terms not having the factor \( \sqrt{7} \times 79 y^3 r^6 \) on LHS of (8) = Nil.

Sum of all rational terms on RHS (8) not having \( \sqrt{7} \times 79 y^3 r^6 \) as a factor:

\[
\left(\left(\sqrt{41 \times 79} y^z z^r \right) \left(41 \ell^{r-1} \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right)
\]

\[
= \left(-7 \times 19 \times 41 \ell^r \left(\sqrt{7} \times 19 \times 79 \ell^z \right)\right)\left(\ell^s \ell^t \right) = \left(-7 \times 19 \times 41 \ell^r \right)
\]

we get

\[
\left(\ell^s \ell^t \right) = \left(7 \times 19 \times 41 \ell^r \right)
\]

That is either \( s = 0 \) or \( t = 0 \). This contradicts our initial hypothesis that all \( r; s; t \) all non-zero integer in the equation \( r^6 + s^6 = t^6 \).

Case (2):

If \( \sqrt{7} \times 79 y^3 r^6 \) is rational:

Sum of such rational terms containing the rational factor \( \sqrt{7} \times 79 y^3 r^6 \) on LHS of (8).

\[
\left(\left(\sqrt{41 \times 79} y^z z^r \right) \left(41 \ell^{r-1} \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right)
\]

\[
= \left(\left(\sqrt{41 \times 79} y^z z^r \right) \left(41 \ell^{r-1} \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right)
\]

\[
= \left(\left(\sqrt{41 \times 79} y^z z^r \right) \left(41 \ell^{r-1} \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right) \left(\sqrt{7} \times 19 \times 41 \ell^{r-1} \ell^r \right)
\]

Sum of such rational terms on (8),

\[
\left(7 \times 19 \times 41 \ell^r \left(\sqrt{7} \times 19 \times 79 \ell^z \right)\right)\left(\ell^s \ell^t \right) = \left(7 \times 19 \times 41 \ell^r \right)
\]
\[ +\left(7 \times 19 \times 41^2 \ell^2\right)\left(z^b s^p \sqrt[2p+1]{t^{2p+1}}\right)\sqrt{7 \times 79 y^{p+p}} \quad \text{(Combining VII & VIII terms)} \]

\[ = \left(7 \times 19 \times 41^2 \ell^2\right)\left(z^b s^p \sqrt[2p+1]{t^{2p+1}}\right)\sqrt{7 \times 79 y^{p+p} \left(7 + \ell^2\right)} \]

\[ = \left(2^n \times 7 \times 19 \times 41^2 \ell^2\right)\left(z^b s^p \sqrt[2p+1]{t^{2p+1}}\right)\sqrt{7 \times 79 y^{p+p}} \quad \left(\because 2^n = 7 + \ell^2\right) \]

This term gets cancelled with LHS term in (8).

III. **Conclusion**

Since (8) in this proof was derived directly from the transformation form of Fermat’s equations for the index 3 and p where p is any prime > 3, the result st = 0, that we have obtained on solving the transformed equations should reflect on the Fermat’s of Equation \( r^p + s^p = t^p \), thus proving that only a trivial solution exists in the equation \( r^p + s^p = t^p \).

The only main hypothesis that we made in this proof, namely, \( r, s \) and \( t \) are non-zero integers has been shattered by the result st = 0 and proves the theorem.

**REFERENCES**

