Application of Asymmetric-GARCH Type Models to The Kenyan Exchange Rates

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***Abstract* — Modelling and forecasting the volatility of a financial time series has become essential in many economic and financial applications like portfolio optimization and risk management. The symmetric-GARCH type models can capture volatility and leptokurtosis. However, the models fail to capture leverage effects, volatility clustering, and the thick tail property of high-frequency financial time series. The main objective of this study was to apply the asymmetric-GARCH type models to Kenyan exchange to overcome the shortcomings of symmetric-GARCH type models. The study compared the asymmetric Conditional Heteroskedasticity class of models: EGARCH, TGARCH, APARCH, GJR-GARCH, and IGARCH. Secondary data on the exchange rate from January 1993 to June 2021 were obtained from the Central Bank of Kenya website. The best fit model is determined based on parsimony of the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Log-Likelihood criterion, and minimisation of prediction production errors (Mean error [ME] and Root Mean Absolute error [RMAE]). The optimal variance equation for the exchange rates data was APARCH (1,1) - ARMA (3,0) model with a skewed normal distribution (AIC = -4.6871, BIC = -4.5860). Volatility clustering was present in exchange rate data with evidence of the leverage effect. Estimated Kenya's exchange rate volatility narrows over time, indicating sustained exchange rate stability.**

***Keywords* — Asymmetric-GARCH; exchange rates; forecasting; volatility.**

I. Background Information

Most investors and financial analysts are concerned about the uncertainty of the returns on their investment assets caused by the variability in speculative market prices, market risk, and business performance instability [1]. Volatility is considered an important concept for many economic and financial applications, like portfolio optimization, risk management, and asset pricing. Volatility refers to the degree of fluctuations of an observed phenomenon over time [2]. In financial economics, volatility is commonly defined as "the (instantaneous) standard deviation of the random Wiener-driven component in a continuous-time diffusion model" [3]. For instance, if the returns of an asset have large swings, it is said to have higher volatility. In turn, common volatility models relate to the "the conditional variance" of the underlying series [4]. As such, modelling and forecasting the volatility of a financial time series has become an essential area in research.

Most financial time series possess volatility and have unique features referred to as stylized financial time series facts, which include the absence of autocorrelations, heavy tails, asymmetry in time scales, volatility clustering, and leverage effect [5]. Linear time series models are not effective for describing the features of a volatility series as they assume the existence of linear dependence in given series [6]. Furthermore, the linear models are built on the homoscedastic assumption which may not necessarily be held in most time-series data as they might be highly time-variant. Time series data like exchange rates usually exhibit volatility clustering resulting in the violation of the homoscedastic assumption of the equality of variance over time [7]. Therefore, non-linear models, for example, symmetric and asymmetric-GARCH type models, have been proposed as suitable models.

Symmetric properties of financial time series modelling were introduced by Engle and Nelson in 1990s [8]. The symmetric-GARCH models capture leptokurtosis and volatility clustering properties. However, the symmetric-GARCH type model fails to model the leverage effect property, a situation when an unanticipated reduction in prices increases anticipated volatility more than an unanticipated growth in the price of similar magnitude [9]. Another problem of symmetric GARCH-type models is that they do not always fully embrace the thick tails property of high-frequency economic time series. Studies have also

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criticized the symmetric GARCH model that despite the innovation the magnitude of change only influences the restricted variance. That is, both past negative and positive fluctuations have the same impact on the current volatility. This is because the conditional variance must be nonnegative. As a result, the parameters are often constrained to be nonnegative [10]. Conclusively, Asymmetric-GARCH type models have the restricted variance only depending on the magnitude or size and not the sign of the underlying shock; the model does not capture the asymmetry effect in financial time series returns data [11].

To address the shortcoming of symmetric GARCH models, asymmetric GARCH type models have been proposed, such as the Exponential GARCH (EGARCH) model by Nelson [12], Glosten, Jagannathan, and Runkle GARCH (GJR-GARCH) model by Glosten *et al*. [13] and the Asymmetric Power ARCH (APARCH) model by Ding et al. (1993), The Threshold GARCH (TGARCH), and the Integrated GARCH (IGARCH) model by [11] are more suitable models because they capture the time-varying variance of such time series [14]. Asymmetry implies that unanticipated bad news increases restricted volatility more than unanticipated good news of similar magnitude [15]. For instance, the EGARCH model uses the natural logarithm to model the restricted variance unlike the GARCH model, which models the variance directly. Thus, parameter boundaries are not required to guarantee a positive restricted variance. Therefore, asymmetric-GARCH-type models tolerate asymmetric effects of positive and negative innovation [16].

Most empirical studies have demonstrated that asymmetric GARCH type models are more robust compared to the symmetric GARCH type models. The stock market has been a common area where volatility models have been useful, for instance, to model the volatility of stock revenues. In Kenya, Wagala *et al*. [17] examined the most efficient model from the symmetric {ARCH(q) and GARCH (p, q)} and the asymmetric GARCH {IGARCH (p, q), EGARCH (p, q), and TGARCH (p, q)} models to fit the Nairobi Securities Exchange Weekly Returns data. Based on the minimisation of Shwartz Bayesian Criteria (SBC), Akaike Information Criteria (AIC), and the Mean Squared Error (MSE), His study findings revealed that the AR-Integrated GARCH (IGARCH) models with student's t-distribution are the best models for modelling volatility in the Nairobi Stock Market data. Petrică and Stancu [18] empirically examined how symmetric GARCH type models (ARCH and GARCH) and the asymmetric-GARCH type models (EGARCH, TGARCH, and PARCH) can capture the volatility of daily returns of EUR/RON exchange ratio. All the asymmetric models were better than the standard ARCH in minimising the volatility prediction errors. The best model for the daily returns of EUR/RON exchange ratio data was EGARCH (2,1), assuming that the data follows a student's t distribution (AIC ≈ 0.7880).

Hasbalrasol *et al*. [19] examined volatility models' accuracy and predictive performance for the monthly Sudanese exchange rate (SDG/USD) return data from 1999 January to 2013 December. The study compared the standard GARCH, Asymmetric GARCH, and ARMA models assuming the non-normal and normal Student distributions. The findings revealed that the asymmetric GARCH type models better fit the Sudanese exchange rate under the non-normal distribution than the normal distribution and improved the overall estimation for determining restricted variance than the GARCH model. The Ding, Granger, and Engle (DEG)-GARCH model assuming the student t- distribution {AIC = -7.844, Bayesian Information Criterion (BIC) = -7.665} was the best fit for the data. The model produced reliable forecasts and adequately estimated the Sudanese pound exchange rate volatility. Besides, the leverage effect in the series was a common stylised fact in most financial series. Overall, the few reviewed studies have a common agreement that asymmetric-GARCH models are better than symmetric-GARCH models. Thus, the current study applies asymmetric-GARCH models to exchange rate data. The following specific objectives guided the study:

1) To fit asymmetric-GARCH type models (EGARCH, IGARCH, APARCH, GJR-GARCH, and TGARCH) to the Kenyan exchange rate data.
2) To identify the best asymmetric-GARCH type model that best fits the Kenyan exchange rate data.
3) To forecast the Kenyan exchange rate using the best asymmetric-GARCH type model.

II. Methodology

A. Research Design

The study used descriptive research design. The descriptive research design explores the stylized properties of financial time series data which includes volatility clustering, negative kurtosis, and excess skewness. The features were determined by the use of data visualization, descriptive, and inferential statistics.

B. Data Collection

The analysis used monthly exchange rate data in Kenya from January 1993 to June 2021 as a convenience sample available during the study. The secondary data was obtained from the Central Bank of Kenya (CBK) website [20]. Kenya's exchange rate is referenced to the US dollar. The series was downloaded in October 2021, and has 342 data points considered adequate for a time series analysis technique. The period is suitable since the sample covers the period when Kenya was already on a flexible exchange rates regime.

C. Data Analysis

The R statistical software [21] was used to analyse the data. Preliminary analysis done included descriptive statistics, trend analysis, and stationarity test. The selected asymmetric GARCH models (GARCH, TGARCH, APARCH, GJR-GARCH, and EGARCH) were fitted to the stationary log-differenced data based on the functions in the RUGARCH [22].

1) Fitting the Asymmetric-GARCH Type Models

The conditional variance of the return series is expressed as a function of constant, past volatility, and past forecast variance in the generalised ARCH model [23], [24]. The maximum likelihood estimator (MLE) method was used to estimate parameters in the model.

a) The GARCH Model

The conditional variance for the generalised GARCH (p, q) model is defined in (1):

$$V_t^2 = \beta_0 + \sum_{r=1}^{K} \alpha_r \, \mu_{t-r}^2 + \sum_{v=1}^{S} \beta_v \, V_{t-v}^2 \tag{1}$$

where $\beta_0 > 0,\ \alpha_i \geq 0, \beta_j > 0$;
V^2: is the restricted variance;
μ^2: Error term referred to as disturbance term;
K is the order of the generalised terms, i.e., the amount of lagged μ^2 terms;
S is the size of the ARCH terms, i.e., the amount of lagged V^2 terms.

Both β_v and α_r are greater than zero, and the component $\sum_{r=1}^{K} \alpha_r + \sum_{v=1}^{S} \beta_v < 1$ to achieve stationary. Additionally, the restraints $\alpha_r \geq 0$ and $\beta_v \geq 0$ ensures that $V^2$is strictly positive [25].

b) The GARCH Model

The exponential GARCH model with its leverage and asymmetry properties in its equation is defined with conditional variance written as in (2):

$$Ln(V_t^2) = \omega + \sum_{v=1}^{S} \beta_v Ln\,(V_{t-v}^2) + \sum_{r=1}^{K} \alpha_r \left\{ \left| \frac{\mu_{t-r}}{V_{t-r}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma_r \frac{\mu_{t-r}}{V_{t-r}} \tag{2}$$

where $V^2 =$ the conditional variance;
$\mu^2 =$ Disturbance error term;
K = the order of amount of lagged μ^2 terms;
S= the order of the number of lagged V^2 terms.

The model is asymmetric in nature because the component, $\frac{\mu_{t-r}}{V_{t-r}}$, is included with coefficient, γ_r. Since the constant is negative, positive returns shockwaves cause less volatility than negative return shocks when other factors remain constant.

c) Glosten, Jagannathan and Runkle GARCH(GJR-GARCH) Model

The GJR-GARCH variance equation is defined by (3).

$$V_t^2 = \omega + \sum_{r=1}^{K} \alpha_r \, \gamma_{t-r} + \sum_{v=1}^{S} \beta_v \, \delta_{t-v}^2 + \gamma_r \, I_{t-r} \gamma_{t-r} \tag{3}$$

where $\alpha\,, \beta, and\ \ \gamma$ are model parameters.
K and S are the lagged orders of the γ and δ_{t-v}^2 terms,
$I =$ Is a dummy variable, also known as the indicator function, and takes the value zero when the parameter γ_{t-r} is negative and one if it is positive. If γ is positive, negative shocks have more impact than positive shocks. The model parameters are assumed to be positive and that $\frac{\alpha+\beta+\gamma}{2} < 1,$ if all leverage coefficient constants are zero, then the GJR-GARCH model becomes the GARCH model.

d) Power GARCH (PGARCH) Model

The PGARCH or APARCH (K, S) has the variance equation as expressed in (4):

$$V_t^{\delta} = \omega + \sum_{r=1}^{K} (\alpha_r |\gamma_{t-r}| - \gamma_r \gamma_{t-r})^{\wedge}\delta + \sum_{v=1}^{S} \beta_v V_{t-v}^{\delta} \tag{4}$$

where $\omega > 0, \delta > 0, \alpha_r \geq 0, -1 < \gamma_r < 1, r = 1, \ldots, K,\ \beta_v \geq 0, v = 1, \ldots, S,\ \alpha\ and\ \beta$ are the normal ARCH and generalised ARCH parameters, γ_i is the leverage effect parameter and δ is the parameter for the power term component.

e) Threshold GARCH (TGARCH) Model

The threshold GARCH model is an extension of the exponential GARCH and the GJR-GARCH model. Its conditional variance in (5)

$$V_t = \omega + \sum_{r=1}^{K} \alpha_r^{(1)} \mu_{t-r}^{(1)} - \alpha_r^{(2)} \mu_{t-r}^{(2)} + \sum_{v=1}^{S} \gamma_v V_{t-v} \tag{5}$$

where $\mu_{t-r}^{(1)}$ = max (e_t, 0), $\mu_{t-r}^{(2)}$= min (e_t, 0) dan e_t = $\mu_{t-r}^{(1)}$ - $\mu_{t-r}^{(2)}$ are the effects of the threshold. K and S are the lagged orders.

f) Integrated GARCH (IGARCH) Model

The IGARCH model is similar to the ARMA model and is a unit-root GARCH model. A key feature of the IGARCH model is that the effect of previous squared shocks $\pi_{t-r} = a_{t-r}^2 - V_{t-r}^2$ for r >0 on a_t^2 is persistent. A variance IGARCH (K, S) model is expressed in (6).

$$V_t^2 = \omega + \sum_{r-1}^{K} \beta_r V_{t-r}^2 + \sum_{v-1}^{S} (1 - \beta_r)\, a_{t-v}^2 \tag{6}$$

where $1 > \beta_r > 0$;
$V^2 =$ the conditional variance;
K is the order of the number of lagged V_{t-r}^2 terms;
S is the order of the number of lagged a_{t-v}^2 terms.

2) Model Selection Criteria

The model selection criteria examine whether the fitted model optimally balances the goodness-of-fit and parsimony. Several evaluation criteria have been developed to assess the model performance of competing models or orders. Some common criterion is the maximum likelihood ratio test of the models, where a model with the highest log-likelihood value is the best [26]. Suppose the competing models do not have equal parameters. In that case, the principle of parsimony applies, such that the best model minimises criteria such as the AIC, and BIC. The best fit model was determined based on parsimony (AIC, BIC, Log-Likelihood criterion) and minimisation of prediction production errors (ME and RMAE). The five metrics are estimated using (7) and (8) [27].

$$AIC = -2\log(L) + 2\log(p+q) \tag{7}$$

where L indicates the likelihood of the data with a certain model, p and q indicate the lagged orders of AR and MA terms, respectively.

$$BIC = -2log\ (L) + \ 2(m) \tag{8}$$

where n and m are the numbers of observations and parameters in the model, correspondingly, and log (L) is the log-likelihood. The best model is the one that minimizes the AIC or BIC while maximizing the log-likelihood.

3) Model Evaluation Criterion

Clement [28] proposes that out-of-sample forecasting ability is the measure for choosing the best model to be adopted to determine the predictive ability of volatility models. The current study used two model selection criteria: the Mean Error (ME) and Root Mean Absolute Error (MAE). If $\delta^2{}_t$ and $\widehat{\delta^2}_t$ represents the actual and forecasted volatility/variance of a series at time t, then; MSE, which measures the average of the squared individual errors, is estimated using (9) [29].

$$ME = \frac{1}{h+1} \sum_{t=s}^{s+h} \widehat{\sigma_t} - \sigma_t \tag{9}$$

where; h is the number of head steps, and s is the sample size.

A model that minimises the MSE value is a better fit for a given data. RMSE is simply obtained by taking the square root of MSE and is estimated using (10).

$$RMAE = \sqrt{\frac{1}{h+1}\sum_{t=s}^{s+h} |\widehat{\sigma_t} - \sigma_t|} \tag{10}$$

The best asymmetric GARCH model is the one that minimizes the ME and RMAE.

4) Residual Diagnostics

An adequate model for forecasting or, in this case, evaluation of volatility should have residuals similar to a series generated from white noise. The simplest way to examine this is to visualize the residuals in a time plot. A histogram superimposed with a density plot was also used to test if the normality assumption holds. For normally distributed residuals, the density curve should be bell-shaped. In addition, the residuals of a given best model must not be autocorrelated. The Ljung-Box (Q) statistic was employed to examine the presence of autocorrelation. The null hypothesis under this test is that there is no serial correlation. The insignificant Q statistic indicates the absence of serial correlation; hence the model fits the data well.

Both symmetric and asymmetric curves differ by the leverage effect. Let ε_t be a measure of shocks; where a positive value of ε_t depicts a positive shock, and vice versa. A standard GARCH model (symmetric) will have a news impact curve that is quadratic, that is, symmetric and centred around $\varepsilon_{t-1} = 0$ (that is; when there is no bad or good news). In that case, positive and non-positive shocks of the same size yield a similar magnitude of volatility. But conventionally, non-positive shocks can cause more volatility than positive shocks of the same magnitude. Thus, the GARCH model underestimates the degree of volatility arising from large negative or bad news/shocks. In the same logic, it overestimates the volatility arising from small positive shocks or good news [8]. To examine the tendencies, three diagnostic tests for volatility models: The Sign Bias Test (SBT), the Negative Size Bias Test (NSBT), and the Positive Size Bias Test (PSBT) are usually carried out. All three tests converge to the evaluation of the model misspecification.

However, reference [8] described, the bias tests have different methodologies. The SBT augments an indicator function S^-_{t-1} which assumes a rate of one if there is negative news ($\varepsilon_{t-1} < 0$) or else zero. It examines whether positive and non-positive shocks affect volatility differently from the fitted model. The NSBT employs a dummy $S^-_{t-1}\varepsilon_{t-1}$. It examines if large negative shocks correlate with the volatility contrary to the fitted volatility model projection. On the contrary, the PSBT uses the dummy variable $S^+_{t-1}\varepsilon_{t-1}$; where $S^+_{t-1} = 1 - S^-_{t-1}\varepsilon_{t-1}$. Unlike the NBST, the PSBT examines how larger positive shocks can impact volatility differently from the forecast of the fitted conditional heteroscedastic model.

Let v_t be the normalized residuals at time t of the fitted volatility model $v_t = \frac{e_t}{\sqrt{h_o}}$. The LM test statistic for $H_0\colon \delta_a = 0$ in any given asymmetric model entails testing the null hypothesis; of $H_0\colon \delta_a = 0$ in the auxiliary regression equation (11).

$$v_t^2 = \underline{Z}_{0t}\underline{\delta}_0 + \underline{Z}_{at}\underline{\delta}_a + \mu_t \tag{11}$$

where; $\underline{\delta}_0$ is the $k \times 1$ direction of parameters of the null hypothesis; $\underline{Z}_{0t}$ is the k x 1 vector of regressors under the null hypothesis; and $\underline{Z}_{at}$ is the $m\ x\ 1$ direction of regressors not included in the model, with associated parameters, $\underline{\delta}_a$, and μ_t is the model residuals. For a perfect or adequate model, the predictors in (9) should be significant. Thus, the conditional heteroscedastic model is misspecified if the fitted model predicts the squared standardised residual. The model residuals should show no ARCH effects; otherwise, they must be modelled. The Lagrange Multiplier (LM) test is employed to test for the existence of ARCH, as was used in the study on each of the best fit models for both exchange rate and BOP. The LM is computed using (12).

$$\xi_{LM} = T \times R^2 \tag{12}$$

where; R^2 is square of the multiple correlation coefficient of (12), and T is the sample size. The LM test statistic takes an asymptotic χ^2 distribution with m as the degrees of freedom, with m denoting the number of restricted parameters. In either case, the null hypothesis is that; there are no ARCH effects. If the resultant p-value associated with either of the statistics is less than 0.05, the null hypothesis will be rejected and vice versa. The regression equations used for evaluation of the SBT, the NSBT, and the PSBT are presented in (13) – (15), respectively.

$$v_t^2 = \alpha + \beta\, S^-_{t-1} + \underline{\beta}'\, Z^*_{ot} + e_t \tag{13}$$

$$v_t^2 = \alpha + \beta\, S_{t-1}^{-}\varepsilon_{t-1} + \underline{\beta}'\, Z_{ot}^{*} + e_t \tag{14}$$

$$v_t^2 = \alpha + \beta\, S_{t-1}^{+}\varepsilon_{t-1} + \underline{\beta}'\, Z_{ot}^{*} + e_t \tag{15}$$

where α and β are parameters of the model, $\underline{\beta}'$ is a vector of parameters associated with the regressors not included in the model, and e_t is a vector of model residuals. The three tests are evaluated based on the t-statistic associated with the coefficient in their respective equations. Alternatively, the three tests can be done jointly following the regression in (16).

$$v_t^2 = \alpha + \beta_1 S_{t-1}^{-} + \beta_2 S_{t-1}^{-}\varepsilon_{t-1} + \beta_3\, S_{t-1}^{+}\varepsilon_{t-1} + \underline{\beta}'\, Z_{ot}^{*} + e_t \tag{16}$$

The t-statistics proportions for β_1, β_2, and β_3 correspondingly, sign bias, negative magnitude bias, and the positive bias test statistics. It is an expectation that, if adequate volatility, then $\beta_1 = \beta_1 = \beta_1 = 0$, $\underline{\beta}' = \underline{0}$; and hence e_t is identically and independently distributed $(i.i.d)$. If coefficients are significant, the positive or negative external shocks affect the variance differently from the model's predictions. On the contrary, there is no bias if the coefficients are insignificant.

5) Forecasting

The ultimate goal of time series modelling techniques is to make future forecasts. Both the in-sample predictions (observed) and out-sample predictions (unobserved) were estimated alongside the one standard deviation confidence band where the actual values are likely to lie. 12-step ahead forecasts from July 2021 to June 2022 were made for each variable.

III. Results And Discussions

A. Trend Analysis

Fig. 1 shows Kenya's exchange rate data from January 1993 to June 2021. The figure shows an overall increasing trend with notable undulation over time. There was a sharp decline around 1992, 2007, and 2012, which can be associated with the usual electioneering period associated with election violence which decreases investor confidence, especially among foreigners. The long periods of exchange rate stability are seen when there is a smooth government transition. For instance, the periods 2003-2005; 2017- 2019 had a relatively stable exchange rate. However, the recent COVID 19 has seen a sharp appreciation in Kenya's exchange rates.

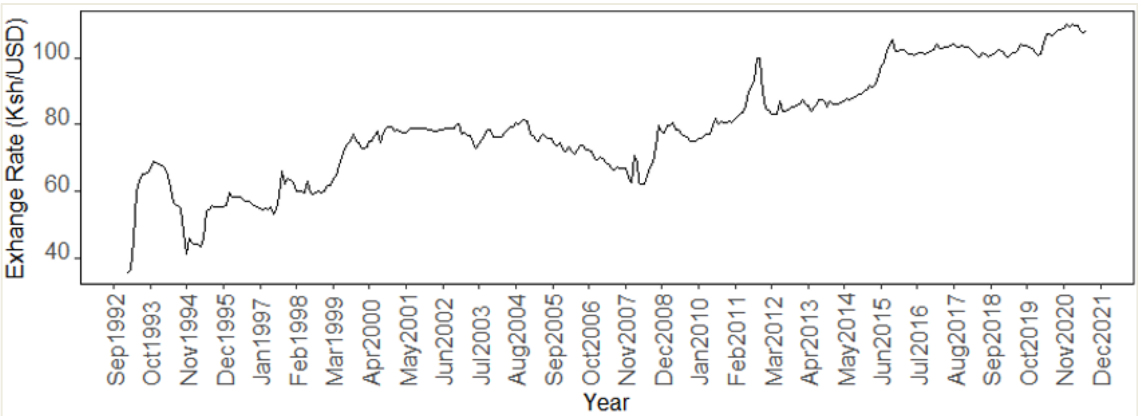

Fig. 1. Trend analysis of exchange rate data.

B. Stationarity Test

The use of non-stationary data in time series analysis has always been criticised since it leads to spurious results. In the current study, we employed the Augmented Dickey-Fuller (ADF) test [30] to evaluate the stationarity of the data. The ADF test tests the hypothesis (H_0) that data is not stationary versus an alternative hypothesis (H_1) which states the data is stationary. The results show that the data is stationary since the p-value is less than p-value at level, at a 5% level (ADF = -3.845, p = 0.044 > 0.05) (Table I). However, it has an upward trend. To obtain data that is stationary, the monthly first difference of the logarithms of the absolute series for exchange rate data was computed using the formula in (17).

$$D.\log(X_t) = \ln|X_t| - \ln\,|X_{t-1}| \tag{17}$$

TABLE I: ADF TEST FOR EXCHANGE RATE DATA

Variable	Series	ADF Statistic	Lag Order	p-value	Comment
Exchange rate	At level	-3.485	6	0.044	Stationary
	Log differenced	-7.555	6	0.010	Stationary

The resultant stationarity series are visualized in Fig. 2

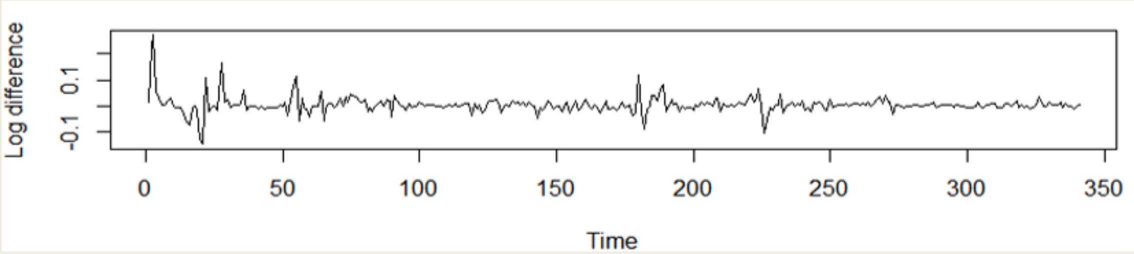

Fig. 2. Time plot of log differenced series for Kenya's exchange rate data.

C. Descriptive Statistics

Table II shows the descriptive statistics and normality test of Kenya's exchange rate data and log-differenced series over the study period.

TABLE II: DESCRIPTIVE STATISTICS AND NORMALITY TEST

		Descriptive Statistics							Shapiro-Wilk test	
Variable	Series	N	Min	Max	Mean	SD	Skewness	Kurtosis	Statistic	p-value
Exchange rate	At level	342	35.92	110.14	79.26	16.49	0.000	-0.599	0.967	0.000
	Log differenced	341	-0.151	0.274	0.0332	0.0332	2.432	24.841	0.7014	0.000

The skewness statistic for the exchange rate data is close to zero, indicating that the data approximately follows a normal distribution. The negative kurtosis (-0.599) suggests that the data is not heavy-tailed; instead, the outliers are less extreme than that of a normal distribution [31]. The low-value kurtosis statistic specifies that the series is slightly platykurtic. However, the log differenced series of the exchange rate data has more interesting properties associated with the stylised facts of the financial series. The series mean is close to zero and has excess kurtosis. As a result, the study proposed that the asymmetric GARCH model can be more appropriate. One of the important features in fitting GARCH-type models is the normality assumption. As statistically quantified by Shapiro-Wilk's (SW) test indicates that all the series are not normally distributed (all $p < 0.05$). The histograms in Fig. 3 indicate that the series slightly deviated from normal.

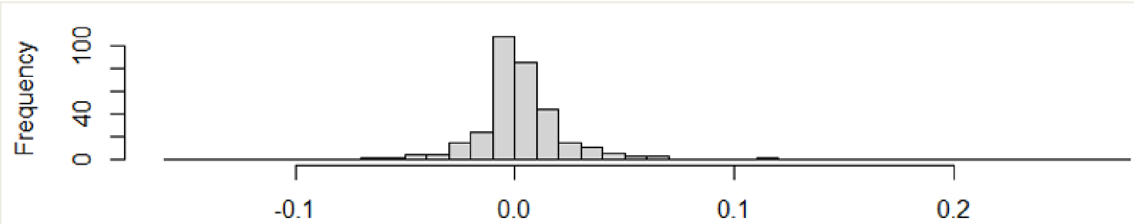

Fig. 3. Histogram of log-differenced series of Kenya's exchange rate data.

Given that the data doesn't follow a standard distribution, mainly caused by the flatter tail than those of a normal distribution, it is necessary to use non-normal distributions which includes: the student t-distribution (std), generalized error distribution ("ged"), and skewed student distribution ("sstd") or skewed normal distribution (snorm) [24], [32]. Given that the data slightly deviates from normal, the current study adopted the $snorm$ when fitting all the potential asymmetry GARCH models for the log-differenced series of Kenya's exchange rate data.

D. Testing for ARCH Effects

Since the GARCH type model is an extension of the ARCH class of model. The GARCH (p, q) model points towards the ARCH ($r = q + p$) model. Thus, the preliminary stage to fitting the model entails testing for the existence of ARCH effects. The Lagrange Multiplier (LM) was employed to examine the presence of ARCH on the squared residuals of the AR (p) model. The null hypothesis state that there is no presence of ARCH properties. The LM test results are shown in Table III.

TABLE III: ARCH EFFECTS TEST FOR THE LOG-DIFFERENCED SERIES OF EXCHANGE RATE DATA

Variable	N	Chi-square Statistic	Degrees of freedom	p-value
Log-differenced exchange rate	342	73.86	12	0.000

Regarding the log-differenced series of exchange rates, the resultant LM statistic at 12 degrees of freedom is 73.86 with an associated p-value less than 0.01. Therefore, the null hypothesis should not be accepted (rejected) at a 1% level of significance, indicating strong evidence of the presence of ARCH effects. The LM was also applied to test for the existence of the ARCH effect on the squared errors of an AR (p) process. The findings justify the use of the GARCH-type models to model the series. The generalized ARCH (GARCH) [24] is an extension of the ARCH family, where σ_t^2 It depends on the lags and lags of the squared error term. The GARCH model is an Autoregressive Distributed Lag (ADL) (p, q) model hence likely to provide more parsimonious parameterisations than the ARCH model.

E. Model Selection and Specification

1) Mean Equation Selection

The mean equation in asymmetric GARCH models is an ARMA process. The mean equation selection for the log differenced series of exchange rates data is discussed below. One of the visualisation tools that help examine the presence of autocorrelation in time series data is ACF; as shown in Fig. 4, there is a significant autocorrelation coefficient with some seasonality, which repeats itself after some interval k.

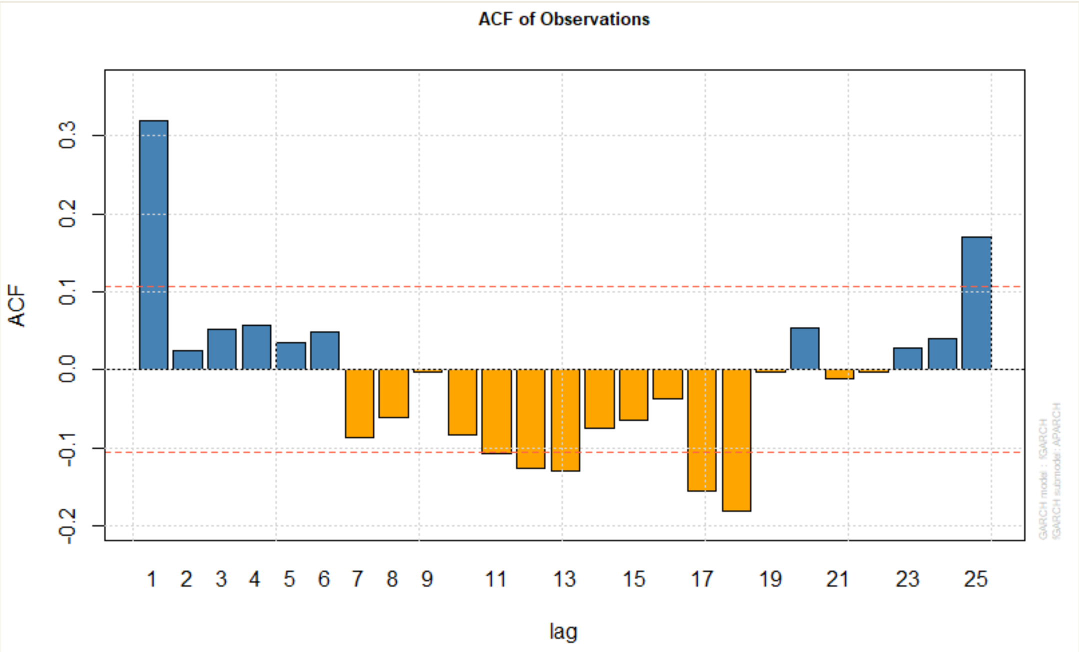

Fig. 4. ACF of log differenced series of Kenya's exchange rate data.

Fig. 5 shows the log differenced series of exchange rate data alongside its decomposed time-series properties and ascertains that the series has seasonality.

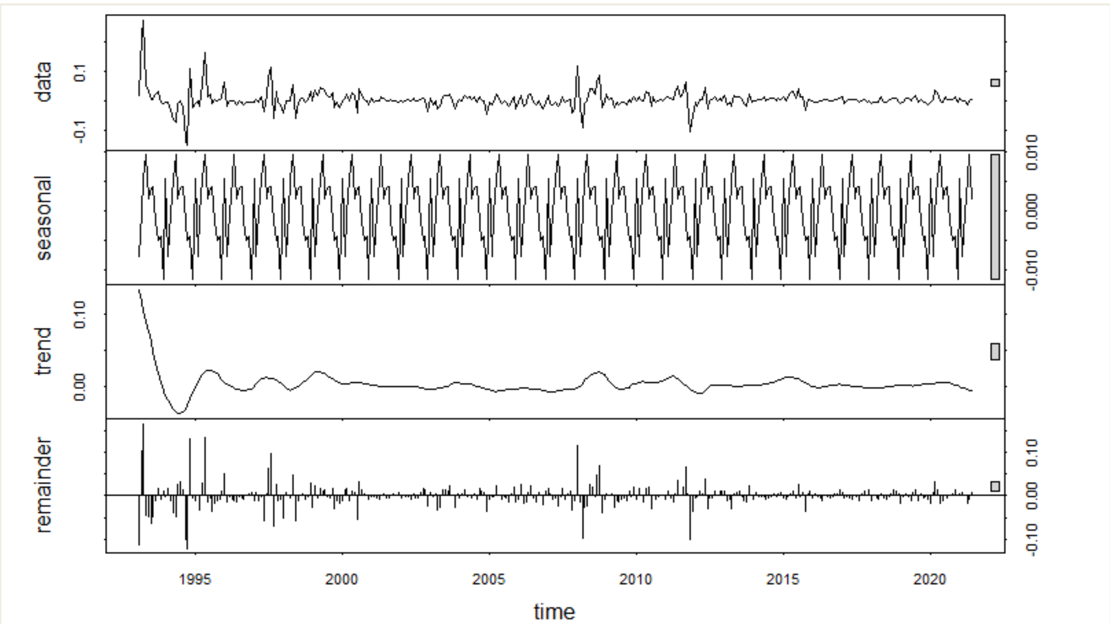

Fig. 5. Time series decomposition of log-differenced series of exchange rates.

One of the notable challenges with modelling with the $ugarchspec$ function in the Rugarch package in R [22] is that it does not account for the seasonality aspect in the ARMA mean equation model. Based on the minimisation of AIC, and BIC values the $auto.arima$ function in the $fpp2$ package in R suggests that ARIMA (4,0,0) (2,0,0) [12] with non-zero mean could be the best Seasonal ARIMA (SARIMA) model (AIC = -1385.34, BIC = -1354.68). The model shows that the mean equation model should capture seasonality. Yet, the RUGARCH package does not account for seasonality in the asymmetric modelling. The seasonal argument was set to False to account for seasonality assuming the series has no seasonal component to incorporate the Fourier terms in the model [33]. The resultant models incorporate the ARMA and Fourier terms as external regressors. K was specified as from 1 to 6 since K should not be greater than period/2. Given that the periodicity of the series is 12; $1 \leq k < 6$. All the ks favoured ARIMA (3,0,0). Therefore, the mean equation in the *ugarchspec* function in the RUGARCH package will be ARMA (3,0). Besides, the best fit GARCH had the parameters p and q as one. The optimal ARMA (3,0) is akin to AR (3) process, indicating autocorrelation in the series.

2) GARCH Model Selection

Table IV presents possible ARMA (3, 0) – APARCH (1, 1) models for the log-differenced series of exchange rate data alongside their evaluation metrics (AIC and BIC). The optimal variance equation based on the minimization of the two metrics was APARCH (1,1) - ARMA (3,0) model with a skewed normal distribution (AIC = -4.6871, BIC = -4.5860).

TABLE IV: MODEL SELECTION FOR THE LOG-DIFFERENCED SERIES OF EXCHANGE RATE DATA

GARCH Model	Mean Model	AIC	BIC	LL	ME	RMAE
EGARCH (1,1)	ARMA (3,0)	-4.4492	-4.3593	766.5814	-0.000603	0.130427
IGARCH (1,1)	ARMA (3,0)	-4.6394	-4.5720	797.0147	0.0034046	0.133446
APARCH (1,1)	ARMA (3,0)	-4.6871	-4.5860	808.1503	0.0019585	0.1340027
TGARCH (1,1)	ARMA (3,0)	-4.1037	-4.0138	707.6726	-0.0054697	0.1422132
GJR-GARCH (1,1)	ARMA (3,0)	-4.6285	-4.5386	797.1539	0.00288466	0.1290308

The findings differ from those of Petrică and Stancu [18], who established that AR (3) - EGARCH (2, 1) was the best model fit to the best model for estimating daily returns of EUR/RON exchange rate. While the mean equation is similar, the GARCH model is different. The findings can be associated with methodological disparity. The current study specified a skewed normal distribution, unlike Petrică and Stancu [18], who preferred students' t-distribution (AIC ≈ 0.7880). Apart from using a different reference currency, the frequency of the series was high (daily EUR/RON exchange rates) and spanned 4th January 1999 to 13th June 2016. Unlike their study, the current study used monthly KES/USD exchange rates which portrayed different time-series properties. In the current study, ARMA (3,0) - APARCH (1,1) is the best fit model since APARCH of Ding, Grange, and Engle [34] accounts for leverage and the Taylor effect [35], which postulates that the observed that the sample autocorrelation of absolute returns was larger than that of squared returns.

Table V summarizes the optimal parameters for the best model ARMA (3,0). - APARCH (1,1). While determining the optimal order, ω was fixed to 0.000020 and was determined based on automatic optimization of the parameter based on the garchFit () function in the Rugarch package [22]. From the results, ϕ_3 of the AR (3) mean equation process is -0.151835 and is statistically significant (p = 0.000369), indicating a negative autocorrelation. Aggregately, the three AR parameters suggest that the exchange rate is characterized mainly by trends, as indicated in the decomposition of the series in Fig. 5. As shown in Fig. 4, the log differenced series of exchange rate data has a decreasing trend component.

TABLE V: OPTIMAL PARAMETERS

Parameter	Estimate	Std. Error	t value	p-value
mu	0.001532	0.00089	1.710710	0.087135
AR1	0.072017	0.076051	0.946951	0.343664
AR2	-0.043664	0.065509	-0.666541	0.505065
AR3	-0.151835	0.042632	-3.561516	0.000369
Omega (fixed)	0.000020			
alpha1	0.649600	0.133069	4.881666	0.000001
beta1	0.206667	0.077380	2.670788	0.007567
eta11	-0.000005	0.071421	-0.000065	0.999948
Lambda	2.475383	0.051397	48.162375	0.000000
skew	1.128859	0.051947	21.731141	0.000000

Note. restricted Variance Dynamics: GARCH Model: APARCH (1,1); Mean Model: ARMA (3,0); Distribution: snorm

Algebraically, the square residuals models can be represented as follows:

$$\sigma_t^2 = 0.000020 + 0.0720X_{t-1} - 0.0437X_{t-2} - 0.15184X_{t-3} + 0.6496\,|\epsilon_{t-1}| + 0.2067\,\sigma_{t-1}^2 - 0.000005v_{j1} + 2.47538\,\epsilon_{t-1}$$

The GARCH parameters were approximated to be α_1= 0.6496, β_1=0.2067, $\eta_{11} = -0.000005$, $\lambda = 2.4754$. A key stylized feature of financial time series data that GARCH models capture is volatility clustering. The persistence parameter captures the feature $\hat{P}$. For the APARCH model, the persistence model is estimated using (18).

$$\bar{P} = \sum_{j=1}^{P} \beta_j + \sum_{k=1}^{q} \alpha_j k_j \tag{18}$$

Given that α_1 and β_1 is statistically significant at a 1% level (p < 0.01), there is a persistent volatility clustering in the series. Regarding volatility persistence, the research findings revealed a small value of the persistence parameter (β) hence a rapid decrease of the rises in the conditional variance due to shocks. Besides, the significance of $\lambda = 2.4754 > 0$ (SE = 0.0514; p < 0.01) suggest that there is a statistically significant leverage effect. That is the null hypothesis (H_0) of no leverage effect ($H_0: \lambda = 0$) was rejected at a 1% significance level. The leverage effect is a common feature in financial time series, where large negative past observations of α_t increases volatility more than positive past observations of a similar magnitude. The non-zero leverage parameters ascertain the presence of asymmetry in the exchange rate series. Conventionally, negative leverage parameters indicate an asymmetric response for positive returns in the conditional variance equation. In contrast, positive leverage parameters indicate that negative shocks or bad news increase volatility [18]. Thus, the resultant positive coefficient of λ (positive asymmetry) shows the absence of leverage result in the exchange rate series. Instead, volatility is positively correlated with the series. In the current finding, positive shocks on the exchange rate generate higher volatility than negative shocks of equal magnitude; other factors are kept constant.

Despite inconsistent results with theory, there is always possible that empirical evidence deviates from the theoretical perspective. For instance, while modelling volatility or return series Nigerian, Onwukwe, Bassey and Isaac [36] found a positive coefficient of λ in the UBA, Mobil, and Unilever returns with returns for the Guinness stock prices only indicating the presence of leverage effect. In another study, Benedict [37] demonstrated the absence of the leverage effect in Ghana's monthly inflation. The study of Nwoye [38] also revealed the existence of volatility clustering, leverage effect, and asymmetric effect in the Exchange rate of Nigeria naira against the USD spanning January 1999 to December 2012, though using the EGARCH (2,2) model as the best model for the data. Petrică and Stancu [18] also found the presence of positive and negative asymmetry in the returns of EUR/RON exchange rate while using the AR (3) - EGARCH (2, 1) model.

Pahlavani and Roshan [39] also established a positive leverage effect in the exchange rate of Iran (IRR/USD) using that ARIMA (7,2), (12) – EGARCH (2,1) was the best fit model. Contrarily, Thorlie, Song, Wang, and Amin [40] found negative asymmetry in the Sierra Leone/USA dollars exchange rate returns computed from the monthly data from January 2004 to December 2013 while using asymmetric GJR-GARCH models. In Kenya, Omari, Mwita, and Waititu [41] found asymmetry and the presence of a negative leverage effect in Kenya's daily exchange rates spanning 3rd January 2003 to 31st December 2015 while using the AR (2)-E-GARCH (1, 1) and AR (2)-GJR-GARCH (1, 1). The findings vary due to the different time frames used to estimate volatility.

F. Residual Diagnostics

The current study tested whether residuals from the best fit model for the exchange rate data meet the normality assumption, show no serial association, and have no ARCH properties. The key aim is to examine whether the model minimizes the prediction errors. Fig. 6 shows the graph of the residuals from the ARMA (3,0)- APARCH (1,1) alongside the histogram. It shows clear evidence that the residuals $\{\epsilon_t\}$ mimics a Gaussian white noise.

Besides, there are few significant autocorrelation coefficients in the resultant model residuals, as depicted in Fig. 7.

In a nutshell, the residuals from the fitted model are purely random; hence the model is adequate. However, randomness is not the only necessary condition that must be satisfied. The residuals of an adequate model should not be autocorrelated. The Ljung–Box test was applied to the model residuals and the squared errors of the best fit ARMA (3,0) -APARCH (1,1) model for Kenya's exchange rate series. The null hypothesis state that there is no serial association. In both cases, the Ljung–Box tests results showed significant results (all p – values < 0.05) for all lags, evidence of the presence of serial correlation (Table VI). However, as shown by the ACF plot in Fig. 7, there are few significant autocorrelation coefficients in the resultant model residuals hence has no serious implication in the fitted GARCH model.

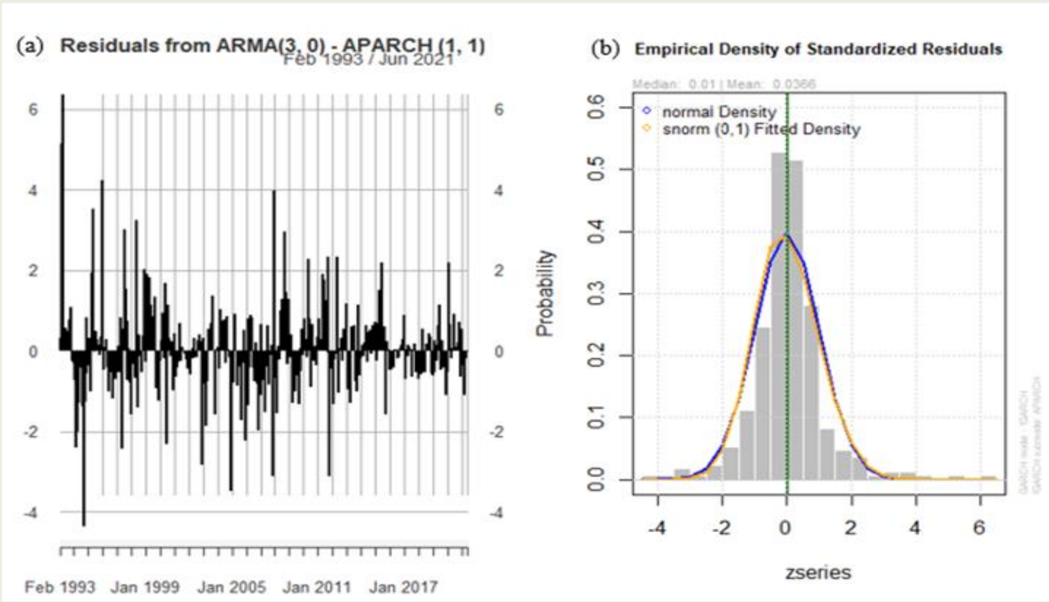

Fig. 6. (a): Residuals plot from ARMA (3,0) -APARCH (1,1) model; (b): the empirical density of standardized residuals.

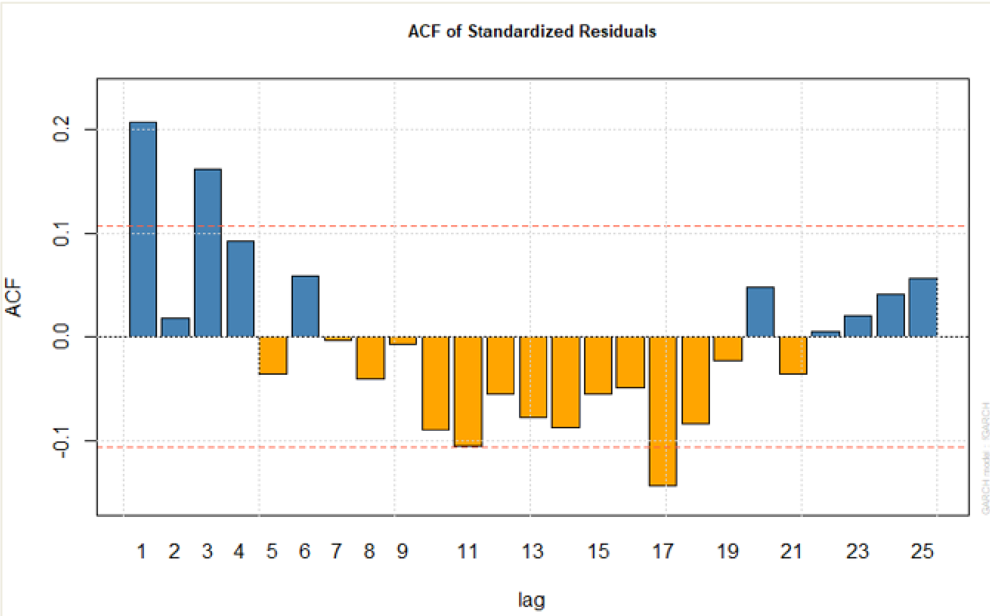

Fig. 7. ACF plot of residuals from ARMA (3,0) -APARCH (1,1) model.

TABLE VI: WEIGHTED LJUNG-BOX TEST

	Standardized residuals		Standardized squared residuals	
Level	statistic	p-value	statistic	p-value
Lag [1]	14.81	0.000	17.50	0.000
Lag[2×(p+q)+(p+q)-1][8]	24.30	0.000	17.87	0.000
Lag[4×(p+q)+(p+q)-1][14]	29.29	0.000	19.20	0.000

Note. degrees of freedom = 3

Furthermore, it is also an expectation that ARCH-LM test results are non-significant. The LM test examines the null hypothesis that "there are no ARCH effects." The results in Table VII indicate the absence of ARCH effects in the model residuals (all p-values > 0.05). Thus, the finding indicates that the fitted ARCH process was adequate.

TABLE VII: WEIGHTED ARCH LM TESTS

Lag order	Statistic	Shape	Scale	P-Value
ARCH Lag [3]	0.2487	0.500	2.000	0.6180
ARCH Lag [5]	0.5832	1.440	1.667	0.8588
ARCH Lag [7]	0.8014	2.315	1.543	0.9436

The SBT, the NSBT, and the PSBT are usually carried out to evaluate the volatility model misspecification. Overall, the SB test evaluates the presence of leverage effects in the standardized residuals (to address the possibility of model misspecification) by regressing the squared standardized residuals on lagged negative and positive shocks [8]. Thus, it tests whether positive and negative shocks affect volatility differently from the fitted conditional heteroscedastic model. The NSBT examines if large negative shocks correlate with the volatility contrary to the fitted volatility model projection. On the contrary, the PSBT examines how larger positive shocks are associated with large biases in forecasted volatility. Hypothetically, the residuals are said to be identically and independently distributed ($i.i.d$) because the model parameters ($\{\beta_i, \underline{\beta}'$) estimated using (13) – (16) are insignificant; hence the model is correctly specified. The fitted model SB and NSB tests are non-significant. However, the PSB is significant and, therefore, the joint effect at a 1% significance level (all p-values < 0.01) (Table VIII). Thus, the squared residuals' significant positive reaction to shocks. Yet, the APARCH model has been designed to alleviate such biases [22].

TABLE VIII: WEIGHTED ARCH LM TESTS

Test	t-value	p-value	Significance
Sign Bias	0.6441	0.052	Not Significant
Negative Sign Bias	0.1416	0.089	Not Significant
Positive Sign Bias	5.0158	0.000	Significant
Joint Effect	26.6334	0.000	Significant

G. *Estimating Volatilities*

While the ARMA (3,0)-APARCH (1,1) model to the log differenced exchange rate data fails the positive sign bias test, the model is adequately supported by the ARCH-LM test; hence can describe the volatility trends of the series. The in-sample volatilities were estimated as graphically presented in Fig. 8. Overall, Kenya's exchange rate volatility appears to be converging over time, indicating sustained exchange rate stability. The GARCH α and β parameters fitted ARMA (3,0) -APARCH (1,1) model were statistically significant at a 1% level ($p < 0.01$), indicating a persistent volatility clustering in the series with rapid decelerating growth over time. The first period depicts some notable occasions where volatility was high. The periods between 1993/94, 1995-1997, 2001/02, 2005/06, 2008/09, 2011-2013, and 2015/17 experienced high volatility and are associated with the electioneering period, which causes an incentive to both foreign and domestic investors due to uncertainty of the election outcome and likelihood of associated violence and disruption of business activities as witnessed in the aftermath of the 2007 elections. Besides, the higher volatility during 2008/09 can also be associated with the global financial crisis. The recent COVID-19 has also seen a sudden spike in the exchange rate volatility during 2020 (Fig. 8).

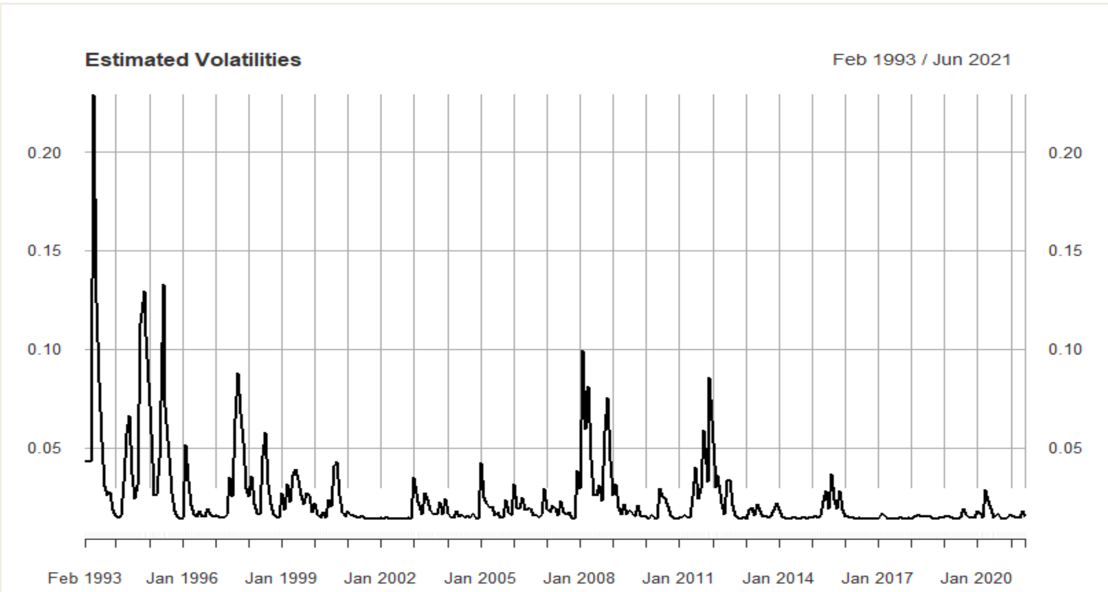

Fig. 8. Estimated exchange rate Volatilities by ARIMA (3,0)-APARCH (1,1).

H. *Forecasting*

The ultimate in time series modelling is to make forecasts. In-sample and out-sample estimates of the exchange rate log differenced series were fisted using the ARMA (3,0)-APARCH (1,1) model having satisfied the model adequacy test. First, an in-sample examination was evaluated where the actual values were superimposed with the estimated one standard deviation from the ARMA (3,0)-APARCH (1,1) model (Fig. 9). The confidence bands portray a similar series' pattern over time, providing reliable future forecasts.

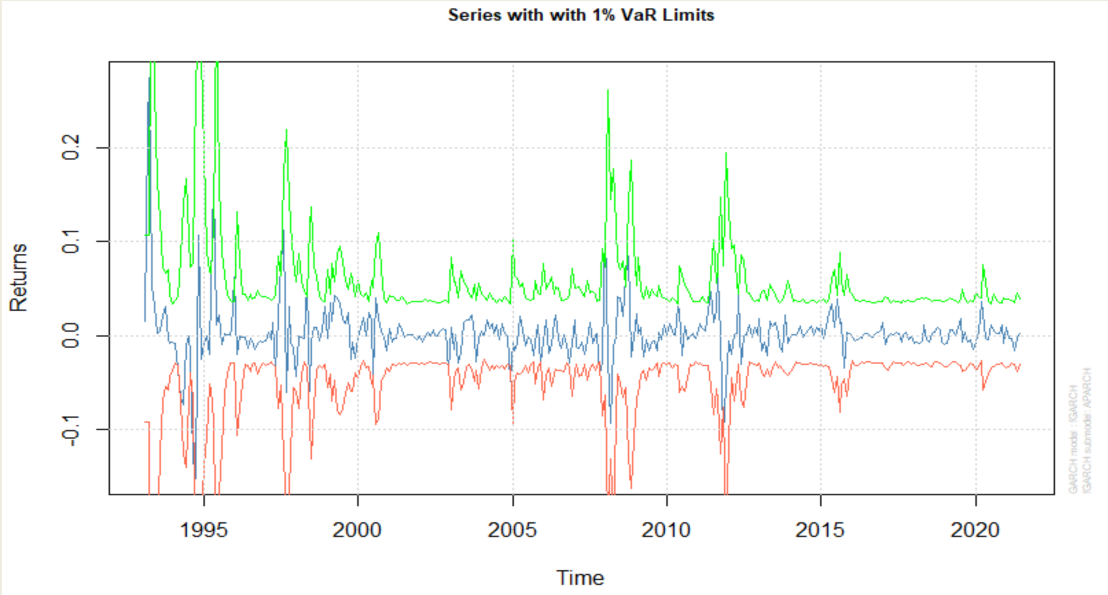

Fig. 9. Actual exchange rate values superimposed with one standard deviation confidence band estimated from ARMA (3,0)-APARCH (1,1).

Fig. 10 plots Kenya's log differenced exchange rates data from Jan 1993 to June 2021 with a 12-month step ahead of July 2021 to June 2022. The 99% confidence limits for the forecasts to account for volatility are included.

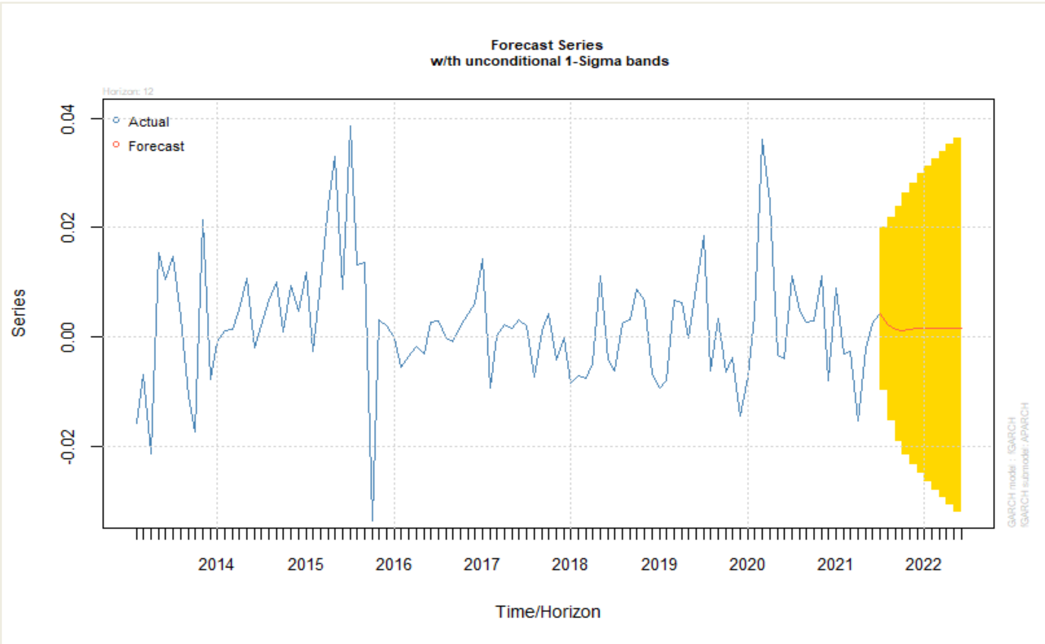

Fig. 10. Log differenced exchange rates (Jan 1992 – June 2021) with a 12-month step ahead forecast with unconditional 1-sigma confidence bands.

The model predicts Kenya's exchange rate to remain relatively constant from July 2021 to June 2022. However, the volatility is increasing over time, as shown by the funnelled shape confident yellow band forecast limits widening out (Fig. 10) due to increasing or diverging conditional standard deviation (σ) over time (Table IX). Thus, while the exchange rate can be somewhat stable and the diverging prediction intervals indicate that it is susceptible to future external shocks, which can either continue to weaken or strengthen it.

TABLE IX: MODEL FORECAST

Forecast	Period	Forecasted Series
T+1	July 2021	0.004329
T+2	August 2021	0.002280
T+3	September 2021	0.001330
T+4	October 2021	0.001060
T+5	November 2021	0.001393
T+6	December 2021	0.001573
T+7	January 2022	0.001613
T+8	February 2022	0.001557
T+9	March 2022	0.001524
T+10	April 2022	0.001518
T+11	May 2022	0.001527
T+12	June 2022	0.001533

Note. Forecast: T0=Jun 2021.

IV. CONCLUSION

The predicted volatilities were captured and are consistent with the shifts in internal and external structural adjustments or shocks. For instance, concerning Exchange rates, volatility was high during the election periods. The recurrent election period in Kenya is a disincentive to investors due to uncertainty about the outcome of the elections. Thus, the electioneering period is linked to higher exchange rate volatility. Operating under the free-float exchange rate, Kenya's exchange rate is susceptible to global economic shocks such as the 2008/09 global financial crisis. The recent COVID-19 has also seen a sudden spike in the exchange rate volatility during 2020. While the exchange rate model will remain stable over the next 12 months, its volatility increases over time. The model's prediction intervals indicate a diverging uncertainty in exchange rates over the next year, signifying a long-run depreciation trend. Besides, the low order is consistent with the stylized fact that economic or financial series are influenced by recent past shocks rather than distant past shocks.

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CONFLICT OF INTEREST

The author declares no competing interests.

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