Research Work of Mathematics in Algebra. New Inventory Work in Quadratic Equation (P.E. Degree 2), Cubic Equation (P.E. Degree 3) & Reducible Equation General Term

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***Abstract* — In this paper, I have introduced new method in Quadratic Equation (P.E.D.2) in order to facilitate the students all over the world to solve these problems in the best way and technique. similarly, I have also invented method in cubic Equation (P.E.D.3) which will enable the students to solve the problems most accurately. Also Reducible Equation general term has been derived which is the base formula in solving P.E.D.2 & P.E.D.3.**

***Keywords* — Basic achievement in Algebra basic, Quadratic Equation i.e. P.E.D.2, Cubic Equation i.e. P.E.D.3, Ahmad Reducible Equation: General Term for n≥2**

I. Introduction

The paper in hand contains two new methods to find Quadratic Equation (Polynomial Equation of degree 2), Cubic Equation (Polynomial Equation of Degree 3) Prior to this new method (P.E.D.2) only one method i. e finding Quadratic Formula by means of completing square was exist and available for the students all over the world and was in practice for more than 500 years. Similarly, there is no authentic and proper formula for solving Cubic Equation (P.E.D.3) exists (Derived) despite 100 of years (500years) past and till date not possible (not derived). I have derived one method for the same Cubic Equation by making it possible which will enable the students to solve the Cubic Equation quite easily & accurately.

Also, Reducible Equation: General term is being invented and introduced which is the key method to solve

Cubic Equation (P.E.D.3), Quartic Equation (P.E.D.4), P.E.D.5, P.E.D.6 and so on. This key method is so important that it made possible to find all the Equations that is Cubic Equation, quartic Equation, Quintic Equation & so on. In addition to this that in the past 500 years or before 500 years i.e. in 1500A.D. the German Scientist in his statement has stated that the solution to solve the problems of polynomial Equation of degree 5 and more than 5 are not possible but I have made it possible by means of reducible equation: General term.

II. Methods

A. Contents

1) *Factorization of Quadratic Equation i.e.* $ax^2+bx+c=0$

Rule: 1. By substitution method

2) *Factorization of Polynomial Equation of Degree 3 i.e. (Cubic Equation) ax3+bx2+cxd=0*

Rule: 1. By substitution method

3) *AHMAD'S Reducible Equation General Term. AHAMD'S Reducible Equation OF Degree N*

Ahmad's R.E.D(N) for all n≥2

III. Factorization of Quadratic Equation i.e. $ax^2 + bx + c = 0$

A. Rule: 1. By Substitution method

PROOF: First we write standard form of Quadratic equation.

$$ax^2 + bx + c = 0$$

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Dividing both sides by coefficient of x^2 i.e. a.

$$\frac{ax^2 + bx + c}{a} = \frac{0}{a}$$
$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = 0$$
$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

Now taking x common from the first two terms involving variable

$$x\left(x + \frac{b}{a}\right) + \frac{c}{a} = 0$$

Then multiplying and divide $\frac{b}{a}$ by 2

$$x\left(x + \frac{2b}{2a}\right) + \frac{c}{a} = 0$$
$$x\left(x + \frac{b+b}{2a}\right) + \frac{c}{a} = 0$$
$$x\left(x + \frac{b}{2a} + \frac{b}{2a}\right) + \frac{c}{a} = 0 \qquad (1)$$

Now substitute $\left(x + \frac{b}{2a}\right)$by any other variable

Let $x + \frac{b}{2a} = y$ (2)

$or\ x = y - \frac{b}{2a}$ (3)

Putting $x + \frac{b}{2a} = y$ and $x = y - \frac{b}{2a}$ in (1)

$$x\left(x + \frac{b}{2a} + \frac{b}{2a}\right) + \frac{c}{a} = 0$$
$$\left(y - \frac{b}{2a}\right)\left(y + \frac{b}{2a}\right) + \frac{c}{a} = 0$$
$$(y)^2 - (\frac{b}{2a})^2 + \frac{c}{a} = 0$$
$$y^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$
$$y^2 + \frac{-b^2 + 4ac}{4a^2} = 0$$
$$y^2 - \frac{b^2 - 4ac}{4a^2} = 0$$
$$y^2 - \frac{(\sqrt{b^2 - 4ac})^2}{(2a)^2} = 0$$
$$y^2 - \left(\sqrt{\frac{b^2 - 4ac}{2a}}\right)^2 = 0$$

Now using formula i.e.$a^2 - b^2 = (\mathrm{a} + \mathrm{b})(\mathrm{a} - \mathrm{b})$

$$\left(y + \frac{\sqrt{b^2-4ac}}{2a}\right)\left(y - \frac{\sqrt{b^2-4ac}}{2a}\right) = 0 \qquad (4)$$

From 2 putting $y = x + \frac{b}{2a}$ in (4)

$$\left(x + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}\right)\left(x + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}\right) = 0$$

Now equate each factor to zero we get

$$x + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} = 0 \ or \ x + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} = 0$$
$$x = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} \ or \ x = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Which is known as Quadratic formula.

IV. Factorization of Polynomial Equation of Degree 3 i.e. (Cubic Equation)

$$ax^3 + bx^2 + cx + d = 0$$

A. Rule: 1. By Substitution method

PROOF: First we write standard form of polynomial equation of degree 3 in one variable.

$$ax^3 + bx^2 + cx + d = 0$$

Dividing both sides by coefficient of x^3 i.e. a.

$$\frac{ax^3 + bx^2 + cx + d}{a} = \frac{0}{a}$$
$$\frac{ax^3}{a} + \frac{bx^2}{a} + \frac{cx}{a} + \frac{d}{a} = \frac{0}{a}$$
$$x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} = 0$$

Now taking x common from the first three terms involving variable

$$x\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) + \frac{d}{a} = 0$$

Again taking x common from the first two terms involving variable

$$x\left[x\left(x + \frac{b}{a}\right) + \frac{c}{a}\right] + \frac{d}{a} = 0$$

Then multiply and divide $\frac{b}{a}$ by 3

$$x\left[x\left(x + \frac{3b}{3a}\right) + \frac{c}{a}\right] + \frac{d}{a} = 0$$
$$x\left[x\left(x + \frac{b + 2b}{3a}\right) + \frac{c}{a}\right] + \frac{d}{a} = 0$$
$$x\left[x\left(x + \frac{b}{3a} + \frac{2b}{3a}\right) + \frac{c}{a}\right] + \frac{d}{a} = 0 \quad (1)$$

Now substitute $\left(x + \frac{b}{3a}\right)$ by any other variable

$$let \ x + \frac{b}{3a} = y$$
$$or \ x = y - \frac{b}{3a}$$

Putting $x + \frac{b}{3a} = y$ and $x = y - \frac{b}{3a}$ in (1)

$$x\left[x\left(x+\frac{b}{3a}+\frac{2b}{3a}\right)+\frac{c}{a}\right]+\frac{d}{a}=0 \quad (1)$$

$$\left(y-\frac{b}{3a}\right)\left[\left(y-\frac{b}{3a}\right)\left(y+\frac{2b}{3a}\right)+\frac{c}{a}\right]+\frac{d}{a}=0$$

$$\left(y-\frac{b}{3a}\right)\left[y^2+\frac{2b}{3a}y-\frac{b}{3a}y-\frac{2b^2}{9a^2}+\frac{c}{a}\right]+\frac{d}{a}=0$$

$$\left(y-\frac{b}{3a}\right)\left[y^2+\frac{(2b-b)}{3a}y-\frac{2b^2}{9a^2}+\frac{c}{a}\right]+\frac{d}{a}=0$$

$$\left(y-\frac{b}{3a}\right)\left[y^2+\frac{b}{3a}y-\frac{2b^2}{9a^2}+\frac{c}{a}\right]+\frac{d}{a}=0$$

$$y^3+\frac{b}{3a}y^2-\frac{2b^2}{9a^2}y+\frac{c}{a}y-\frac{b}{3a}y^2-\left(\frac{b}{3a}\right)^2y+\frac{2b^3}{27a^3}-\frac{bc}{3a^2}+\frac{d}{a}=0$$

$$y^3-\frac{2b^2}{9a^2}y+\frac{c}{a}y-\frac{b^2}{9a^2}y+\frac{2b^3}{27a^3}-\frac{bc}{3a^2}+\frac{d}{a}=0$$

$$y^3+\frac{(-2b^2+9ac-b^2)}{9a^2}y+\frac{(2b^3-9abc+27a^2d)}{27a^3}=0$$

$$y^3+\frac{(-3b^2+9ac)}{9a^2}y+\frac{2b^3-9abc+27a^2d}{27a^3}=0$$

Multiply both sides by $27a^3$

$$27a^3\left[y^3+\frac{(-3b^2+9ac)}{9a^2}y+\frac{2b^3-9abc+27a^2d}{27a^3}\right]=27a^3(0)$$

$$27a^3y^3+27a^3\frac{(-3b^2+9ac)}{9a^2}y+27a^3\frac{(2b^3-9abc+27a^2d)}{27a^3}=0$$

$$27a^3y^3+3a(-3b^2+9ac)y+(2b^3-9abc+27a^2d)=0$$

$$(3ay)^3+(-3b^2+9ac)(3ay)+(2b^3-9abc+27a^2d)=0$$

$$(3ay)^3-3(b^2-3ac)(3ay)+2b^3-9abc+27a^2d=0 \quad (2)$$

Now substitute $3ay$ by any other variable

Let $3ay=P$

Putting$3ay=P$ in (2)

$$(3ay)^3-3(b^2-3ac)(3ay)+2b^3-9abc+27a^2d=0 \quad (2)$$

$$(P)^3-3(b^2-3ac)(P)+2b^3-9abc+27a^2d=0$$

$$P^3-3(b^2-3ac)P+2b^3-9abc+27a^2d=0 \quad (3)$$

Which is known as Ahmad's Equation of Polynomial of degree 3 as given below
Now using Ahmad's Reducible Equation of degree 3 as given below:

$$P^3+(-l^2+mn)P+lmn=0$$

$$or\ P^3+(-A^2+B)P+AB=0$$

$$P^3-A^2P+BP+AB=0$$

$$P(P^2-A^2)+B(P+A)=0$$

$$P(P+A)(P-A)+B(P+A)=0$$

$$(P+A)[P(P-A)+B]=0$$

$$(P+A)[P^2-AP+B]=0 \quad (4)$$

Now equate each factor (i.e. one is linear factor and other is quadratic factor) to zero.

$$P+A=0\ \ldots..eq(5)\ or\ P^2-AP+B=0 \quad (6)$$

After solving $eq(4)$, it is reduced to two equations i.e. linear equation (5) and quadratic equation (6)
From $(5) P+A=0$
Putting $P=3ay$ in (5)

$$3ay+A=0$$

Now putting back $y=x+\frac{b}{3a}$ in the above equation we get.

$$3a\left(x+\frac{b}{3a}\right)+A=0$$
$$3ax+3a\frac{b}{3a}+A=0$$
$$3ax+b+A=0$$
$$3ax=-b-A$$

Dividing both sides by $3a$

$$\frac{3ax}{3a}=\frac{-b-A}{3a}$$
$$or\ x=\frac{-b-A}{3a}\ \ldots\ldots(Va-i)$$

From (6) $P^2-AP+B=0$

The above equation is the standard form of Quadratic Equation.

Now in order to find the values of P from (6), we use either quadratic formula or any one of the 6 Rules. After solving above quadratic equation i.e. (6) we get product of two linear equations

$$i.e.(P+u)(P+v)=0$$

Then equate each factor to zero.

$$P+u=0 \quad (7)$$

$$or$$

$$P+v=0 \quad (8)$$

Putting $P=3ay$ in eq (7) and $eq(8)$

$$3ay+u=0\ \ or\ \ 3ay+v=0$$

Now putting back $y=x+\frac{b}{3a}$ in both the above equation we get

$$3a\left(x+\frac{b}{3a}\right)+u=0\ \ or\ \ 3a\left(x+\frac{b}{3a}\right)+v=0$$
$$3ax+3a\frac{b}{3a}+u=0\ \ or\ \ 3ax+3a\frac{b}{3a}+v=0$$
$$3ax+b+u=0\ \ or\ \ 3ax+b+v=0$$
$$3ax=-b-u\ \ or\ \ 3ax=-b-v$$

Dividing both the sides of both the equation by $3a$

$$\frac{3ax}{3a}=\frac{-b-u}{3a}\ \ or\ \ \frac{3ax}{3a}=\frac{-b-v}{3a}$$
$$x=\frac{-b-u}{3a}\ \ldots.(Va-ii)\ or\ \ x=\frac{-b-v}{3a}\ \ldots.(Va-iii)$$

From$(Va-i)$, $(Va-ii)$ and $(Va-iii)$ we have

$$x=\frac{-b-A}{3a},x=\frac{-b-u}{3a},x=\frac{-b-v}{3a}$$
$$x=\frac{-b-A}{3a},\frac{-b-u}{3a},\frac{-b-v}{3a}$$

This method of factorization of $ax^3+bx^2+cx+d=0$ by substitution Method is known as Ahmad's Rule.

V. Ahmad's Reducible Equation General Term: Ahmad's Reducible Equation of degree n [Ahmad's R. E. D(n)] For all $n\geq 2$

The General term of Ahmad's Reducible Equation is given by

$$(P+A)[P^{n-1}-AP^{n-2}+BP^{n-3}+CP^{n-4}+DP^{n-5}+\_\_\_\_\_]=0$$
$$P^{1+n-1}-AP^{1+n-2}+BP^{1+n-3}+CP^{1+n-4}+DP^{1+n-5}+AP^{n-1}-A^2P^{n-2}+ABP^{n-3}+ACP^{n-4}+ADP^{n-5}+\_\_\_\_\_=0$$
$$P^{n}-AP^{n-1}+BP^{n-2}+CP^{n-3}+DP^{n-4}+AP^{n-1}-A^2P^{n-2}+ABP^{n-3}+ACP^{n-4}+ADP^{n-5}+\_\_\_\_\_=0$$
$$P^{n}+BP^{n-2}-A^2P^{n-2}+CP^{n-3}+ABP^{n-3}+DP^{n-4}+ACP^{n-4}+ADP^{n-5}+\_\_\_\_\_=0$$
$$P^{n}+P^{n-2}(B-A^2)+P^{n-3}(C+AB)+P^{n-4}(D+AC)+ADP^{n-5}+\_\_\_\_\_=0$$

Which is known as the General term of Ahmad's Reducible Equation.

It is also called Ahmad's Reducible Equation of degree n or Ahmad's R.E.D.(n)

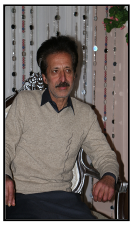
Since the given equation is the Polynomial Equation of Degree n and we know that the degree of the Polynomial Equation is always positive.

VI. Conclusion

This method will be a great achievement in field of Mathematics Algebra and a great addition in this modern age of time. It will also stimulate the thought of interest of the students all over the world.

Conflict of Interest

Authors declare that they do not have any conflict of interest.



A. Raza, was born on 15.01.1967 in Peshawar, Pakistan. He obtained B.S.C degree in the field of Mathematics & English from the university of Peshawar, K.P. His ambition and aim were to get maximum possible Education i.e. to get P.H.D in Mathematics but due to financial constraint was not able to continue and complete his education.

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After getting D.A.E from Government College of Technology Peshawar. He had started his research work in the field of Mathematics, Since January 2009 and still in progress till to date. He had succeeded in deriving six new formulas i.e. six new methods to find the quadratic formula by means of six rules and out of which one of the rule has been included in the manuscript for publication. Beside these six rules, two new methods have also been derived in order to find Cubic Equation and also one of the rules has been included in manuscript for publication. Author has also invented two new rules for quartic equation (P.E.D.4) which will later on be submitted for publication. He is also doing research work in trigonometry and been succeeded in deriving some new formulas.