

An Introduction to Linear Programming Problems with Some Real-Life Applications

R. Kunwar, H. P. Sapkota

Abstract — Linear programming is a mathematical tool for optimizing an outcome through a mathematical model. In recent times different mathematical models are extensively used in the planning of different real-life applications such as agriculture, management, business, industry, transportation, telecommunication, engineering, and so on. It is mainly used to make the real-life situation easier, more comfortable, and more economic, and to get optimum achievement from the limited resources. This paper has tried to shed light on the basic information about linear programming problems and some real-life applications. It presents the general introduction of the linear programming problem, historical overview, meaning and definition of a linear programming problem, assumptions of a linear programming problem, component of a linear programming problem, and characteristics of a linear programming problem, and some highlights of some real-life applications.

Keywords — Constraints, decision variables, linear programming, objective function, optimization.

I. INTRODUCTION

Linear programming (LP) is a branch or subset of mathematical programming. LP is the quantitative analysis technique for deciding to achieve the desired goal. It is a simple optimization technique. It is considered to be the most important method of optimization in different fields. It is used to obtain the most optimal solution to the problem within some constraints. It comprises of an objective function; linear inequalities subject to some constraints may be in the form of linear equations or in the form of inequalities. This method is used to maximize or minimize the objective function of the given mathematical model comprising the set of linear inequalities depending upon some constraints represented in the linear relationship.

LP is concerned with the maximization or minimization of a linear objective function in many variables subject to linear equality and inequality constraints [1]. It deals with the optimization of linear functions subject to linear constraints. Particularly, LP helps to allocate the resources efficiently to profit maximization, loss minimization, or to utilize the production capacity to the maximum extent [2]. LP always helps formulate the real-life problems into a fixed mathematical model. Thus, it is also defined as a mathematical technique for solving different real-life problems to determine the optimal solution or for calculating the best value within the context or situation [3]. It is used to find the optimum utilization of the resources at the minimum cost. In another words, it deals with the allocation of resources with some restrictions such as costs and availability.

It uses some assumptions while determining the optimal value. The linear inequalities or restrictions known as constraints of LP problems are always articulated in quantitative terms. The objective function says the linear function $Z = ax + by$, where a, b are constants and the constraints i.e. $x \geq 0, y \geq 0$ should always be linear and the linear function is to be optimized. LP is extensively used in mathematics and other different fields of social and physical sciences. For instance it is used to make the decision on business planning, industrial engineering, economics, telecommunication, energy, transportation and routing, fields of manufacture, various types of scheduling, etc. It is the simplest and most extensively used method of problem optimization [3]. In a real-life situation, the multi-sector optimization problem usually occurs such as the planning of regional development, development of water or electricity systems, urban planning, and preservation of the natural environment [4].

This article mainly discusses on the historical overview, meaning, and definition of the LP problem, some assumptions, major components, characteristics of LP, and some real-life applications.

A. Historical Overview

The development of the solving LP problem does not have so long history. It is considered to be a revolutionary development in the sense of making optimal decisions in complex situations [5]. The first

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start-up for solving optimization problems with simple equality constraints was done by Lagrange in 1762 [6], [7]. Similarly, Gauss solved linear equations in 1820 [7]. A French mathematician Jean-Baptiste Joseph Fourier (1768-1830) is known as the first man to publish a book of method for solving systems of linear inequalities in 1827 [8]. This was the first attempt at the solution of an optimization problem. Similarly, the Russian mathematician Leonid V. Kantorovich (1912-1986) gave linear optimization formulations of resource allocation problems in 1939 that were used in World War II to reduce the costs and increase the efficiency of the army on the battlefield. In the meantime, the American economist Tjalling C. Koopmans (1910-1985) also contributed to formulating the linear optimization problem.

Similarly, Frank Hitchcock formulated the transportation problem in 1941 and George Stigler formulated the diet problem in 1945 [9]. The real practice of the optimization problem was implemented During World War II by the U.S. Army to allocate the resources effectively in the leadership of George B. Dantzig in 1947 [8]. As a result, both Kantorovich and Koopmans were awarded The Nobel Prize in economic sciences for their work in 1975. Dantzig also developed the linear programming model for solving military logistics problems. After the development of the digital computer in 1945, the American mathematician, George Dantzig developed the Simplex method for solving LP problems in 1951 [9], [6]. For this achievement, George Dantzig was awarded the National Medal of Science by President Gerald Ford considered as the inventor of LP problems in 1976.

The Simplex method was considered as a simplified form to solve the linear programming problems. However, when more variables were attempted, the number of necessary operations expanded exponentially and the computation becomes more complicated. Thus, in 1979, Leonid Khachiyan, the Russian mathematician, discovered a polynomial-time algorithm called the ellipsoid method. This process is considered slower than the Simplex method when practically applied. Likewise, Narendra Karmarkar, an Indian mathematician discovered another polynomial-time algorithm in 1984 called the interior point method [6]. This method was proved as competitive with the Simplex method. In 1968, Fiacco and McCormick introduced the Interior Point Method in 1968. Similarly, in 1984, Karmarkar applied the Interior Method to solve linear programming problems [6].

Gradually, progress was made to a large extent in the theoretical development and the practical applications of the LP problem. John von Neumann recognized the significance of the concept of duality and later Kuhn and Tucker contributed to the theoretical development of the duality theory that is considered as the most important impact on the development of LP problems [9]. Similarly, the works of Charnes and Cooper have also had a major impact on the industrial applications of the LP problem. Similarly, as time passed, other several methods for solving LP problems have been developed. At this time, the method developed by Karmarkar in 1984 was faster in comparison to the simplex algorithm of Dantzig [9]. Thus, the development of different techniques for calculating LP problems helped to solve the decision-making process effectively and efficiently. On the other hand, the actual techniques for solving LP problems have been successfully applied in different sectors such as educational, industrial, and civil.

The development of computers also helps to make it possible to develop for solving the LP problem, called the simplex algorithm at a faster rate. This also helps to develop linear optimization problems rapidly in both theory and its application. Nowadays, with the development of computer software, linear optimization problems having several variables and constraints can be solved in a few minutes. In the present time, linear optimization problems are almost used in academic, industrial, and civil services for quantitative decision-making.

B. Meaning and Definition of Linear Programming Problem

Linear programming is considered to be extremely significant tool due to its application in modeling real-world problems and its use in mathematical theory. Linear programming problem is the specified linear function that is considered as an objective function that may contain several variables subject to the conditions satisfying a set of linear inequalities known as linear constraints and it is concerned with finding the optimal value of the given linear function. A linear function or the objective function has the form: $a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + \dots + a_n x_n = 0$, whereas, a 's are called the coefficients or sometimes called parameters of the equation and x 's are called the variables of the equation. The optimal value can either be a minimum or maximum. The LP problem is used to find the best answer for a range of circumstances, together with manufacturing issues, transportation issues, diet issues, allocation issues, etc.

Different areas of applied mathematics were developed in the late 1940s to solve the allocation problems of the number of resources. An LP problem can be defined as the problem of maximization or minimization of the linear function subject to the linear constraints. The constraints may be equalities or inequalities. It is a special and extremely important type of optimization problem due to its wide applicability in different sectors [10].

The term 'linear programming' comprises of two words. The first word 'linear' describes the relationship between multiple variables with degree one or describes the linear relationship among variables in a specific model [11]. Such a relationship always exists a proportional change to one another variable. This means that the change of one variable always affects the other variable. The second word 'programming' denotes the mathematical modeling to solve the problem using limited resources to achieve the desired objectives [11]. Similarly, programming describes the process of getting the best solution from a variety of alternatives. The term "programming" is also used as a synonym for optimization [12]. Similarly, it is a mathematical technique useful for allocation of 'scarce' or 'limited' resources, to several competing activities on the basis of a given criterion of optimality [11]. According to [13], linear programming is a planning technique that permits some objective functions to be minimized or maximized within the framework of given situational restrictions. LP is a powerful technique of mathematical programming extensively used in business planning, transportation, engineering design, airline scheduling, and many other applications [14]. According to [15] linear programming can be defined as a mathematical technique for solving management problems. While solving LP problems, at first the real-life problems should be modeled in the form of a linear programming model consisting of a certain number of resources into linear equations or inequalities that permit us to appropriately identify and structure the constraints satisfied by the variables of the model. Then, we can calculate the mathematical optimum value using specific linear programming techniques.

II. ASSUMPTIONS OF LINEAR PROGRAMMING PROBLEM

Assumptions are the acceptance without any proof or acceptance without proof. In the LP problem, some assumptions are made to model the complex real-world problem into a simple form that can be solved or analyzed more freely. According to [11], the major assumptions of the LP problem are stated as:

A. Certainty

The assumption of LP, certainty denotes the availability of all the parameters such as the parameters of objective function coefficients and the coefficients of constraint inequalities. In other words, the availability of all parameters for instance: availability of resources, contribution per unit of decision variable, and consumption of resources per unit of decision variable must be known and must be constant.

B. Additivity

Additivity is the value of the objective function and the total amount of each resource used or supplied, must be equal to the sum of the respective individual contribution (profit or cost) of the decision variables. In another word, the sum of all activities equals the sum of each individual activity.

C. Linearity or Proportionality

It is the amount of each resource used and its role to the profit or cost in objective function must be proportional to the value of each decision variable. In other words, any change in the constraint inequalities also makes the proportional change in the objective function is the proportionality or linearity.

D. Divisibility or continuity

The assumption divisibility or continuity denotes the state of continuous decision variables. It means a combination of outputs can be used with the fractional values along with the integer values. It means that the solution needs to be in whole numbers (integers). Instead, they are divisible and may take any fractional value, if the product cannot be produced in fraction, and integer programming problem exists.

III. COMPONENT OF LINEAR PROGRAMMING PROBLEM

The LP problem consists of different components. According to [11], the major components of LP are stated as:

A. Decision variables (activities)

Decision variables are the physical quantities used to be calculated and controlled by the decision-maker and represented by mathematical symbols. These variables are the physical quantities and the decision-maker has control over them. Such variables are usually denoted by x_1, x_2, \dots, x_n , where n is a finite positive integer i.e. $x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0$. These decision variables are usually interrelated in terms of the consumption of resources, and they require simultaneous solutions. In the LP model, all decision variables are continuous, controllable, and nonnegative. These are unknown quantities and are expected to be estimated as an output of the LP problem.

B. The objective function

The objective function of each LP problem is expressed in terms of decision variables to optimize the criterion of optimality. The objective function $Z = c_1x_1 + \dots + c_nx_n$ is a linear function of the n decision variables. The constants c_1, c_2, \dots, c_n are real numbers that are called the coefficient of the objective function. The objective function can be written depending on maximization or minimization. $\text{Max } Z = c_1x_1 + \dots + c_nx_n$ or $\text{min } Z = c_1x_1 + \dots + c_nx_n$ respectively. The value of the objective function at a point x is called the objective value at x . The objective function always specifies a way of optimization, either to minimize or maximize. In the linear function, $Z = c_1x_1 + \dots + c_nx_n$, where c_1, c_2, \dots, c_n are constants, which has to be maximized or minimized is called a linear objective function.

C. The constraints

Constraints are the limitations about the use of resources regarding physical, financial, legal, technological, ethical, and other restrictions that limit the degree to which an objective can be achieved. For instance, the resources, such as raw material, labor, machine, money, manpower, space, etc. may affect the result. Such constraints must be expressed as linear equalities or inequalities in regard to decision variables. The conditions in the given objective function, $x \geq 0, y \geq 0$ are called non-negative restrictions and the solution of an LP problem must satisfy these constraints.

D. Non-negativity

It is the most valuable part of the LP problem that makes sense of the decision variable. Non-negativity of an LP model is an inequality of the form $x_i \geq 0$, and a non-positivity constraint is of form $x_i \leq 0$. It may also happen that a variable x_i is not restricted by a non-negativity constraint or a non-positivity constraint. In such a case, x_i is known as a free or unrestricted variable.

IV. CHARACTERISTICS OF LINEAR PROGRAMMING PROBLEM

The LP problems also have some special features or quality so that the LP technique can be applied. Some of the general characteristics have been already discussed in the assumption and component of the LP problem. The chief characteristics of LP in standard form have been discussed here. The LP problem has a well-defined objective function or the function of optimization and that can be stated in a quantitative way. The decision variables are continuous and they can accept any fractional or non-negative values within the particular range. The constraints or limitations should be articulated in the mathematical form concerning the resource. The variables in the function should be linear. The variable value should be positive or zero. It should not be a negative value. The number of inputs and outputs in LP needs to be finite. Thus LP problem consists four basic characteristics such as objective function, constraints, linearity, non-negativity, and finiteness.

V. SOME REAL-LIFE APPLICATIONS OF LINEAR PROGRAMMING

As described earlier, an LP technique is used for the utilization of resources for the appropriate decision-making. So, it is used to select the best feasible approach from a number of options. According to [11], [16], LP problems are mainly concerned with two important classes of objects. The first is related to the limited resources such as land, labor, capital, materials, machines, etc., and the second is activities related to production such as the production of low carbon, production of more stainless steel, and production of high carbon steel etc. Here, the problem is to decide the best combination of each activity that does not use more resources but produces the best results. So, LP is considered to be a powerful tool for selecting alternatives in a decision making and, accordingly, it has been applied in a wide variety of problems in a real-world situation [11]. LP has a major role to instigate the development of different optimization theories. It is used as a significant tool in different subjects such as economics, management, science, engineering and information communication technology, and other various subjects. Some of the major areas of applications concerning the real-world situation have been discussed below.

A. Agriculture

In the context of agriculture, firstly, LP can be applied in agricultural production planning, farm and land management, distribution of limited resources such as land, labor, money, materials, water supply, etc. in terms of economic perspective. The agricultural production also requires the maximum use of LP that the product should be marketable in terms of the competitive market price. Thus, it should be utilized optimum allocation of crop production, efficient production patterns, optimum use of resources, and a more efficient product distribution system through the use of a linear programming model.

B. Industry

LP can be used in different sectors of various industries. The industry uses LP, especially in the manufacturing goods and service industry. Manufacturing industries use LP to maximize efficiency with minimum operation cost or maximum product in minimum cost. The manufacturer can use the LP model to reconfigure their storage layout, adjust their workforce and reduce the blockage so that they can maximize the production or minimize the cost of the product. LP is used for retail shelf space optimization as a marketing strategy and finding the best delivery routes optimization. The best delivery route minimizes the time and operation cost. In recent times, sales operating costs can be minimized through the use of machine learning software. It is a mathematical model of a function that is systematized through software that can read the value of the labeled input data fast as well as accurately.

C. Engineering

LP is the major tool for engineers in shape optimization that reduces cost and increases efficiency. It helps them solve manufacturing problems and create different designs. It helps increase the production levels for maximizing profit in particular conditions. It also helps solve the problem related to design and manufacturing. Likewise, it assists to increase efficient production and maximize profit. It also helps optimize the electric power system, transportation optimization, water supply, sanitation and irrigation, networking and connectivity, designing, and so on, and to optimize cost and time.

D. Management

Management is the art of smooth and effective running of the system of any organization. It comprises of the systematic process of planning, organizing, leading, controlling the human resources, financial, physical, and information and communication aspects of an organization to get the optimum goals of that organization. Management is linked with different sectors such as finance, production, marketing, personnel, human resource, distribution, etc. Thus, LP is linked with the huge family of management aspects of the organization.

Financial management belongs to the investment to the organization that is intended to maximize the expected return or profit and minimize the risk. Production management is concerned with the use of limited production resources or raw materials, minimum operating costs and time, and it maximizes the total contribution subject to all the constraints. LP techniques also help to determine maximization of the marketing management through effective exposure with the minimum budget. It means with a minimum advertising cost, we want to get the most desirable result. In the same way, LP is used effectively in personnel management. It is used to address staffing problems, determination of equitable salaries, and select a suitable person for a specific job. LP is used effectively in human resource management too. If the human resource is managed as right man in the right place, the effectiveness of the service through the skilled personnel becomes more fruitful and gets the best result within a limited time. It helps to get optimum results from the minimum input or resources. Distribution management also requires LP for more effective distribution of the goods or products. It is the process of managing the movement of goods from supplier or manufacturer to point of seller. So the profit depends on the successful distribution management so that the more they sell, the more they earn. Similarly, several management aspects can be made for the efficient optimization of real-life problems.

E. Transportation

LP problem is effectively used in transportation problem. The transportation problem helps to reduce the cost and maximize the profit if the actual allocation is made. The transportation problem in airlines consists of scheduling crew, fleet, and personnel, root allocation, passenger mix, etc. Similarly, transportation in other vehicles means vehicle routing, transport vehicle scheduling, public transportation schedules, etc that actually reduces the cost, saves time, and maximizes the profit.

F. Government and Public Service

LP problem can be effectively used in government and public services. Especially in security forces and other public services, LP is used effectively. It is used to select different weapons and minimize fuel and vehicles for the security forces. Likewise, transfer scheduling of the officials in different places, providing logistic supports and manpower development plan, LP program can be used effectively to reduce the cost and to provide effective service delivery.

G. Miscellaneous

The LP program can be used in various sectors of daily life. It can be used in pollution control models; sales force deployment, utilizing the storage and distribution centers, proper production scheduling and inventory control, blending problems, minimizing wastage of raw materials, forest and land management, forest valuation models, planting and harvesting models, pattern layout and cutting optimization in the

textile industry, production scheduling, etc. Thus, LP can be applied to various other fields of planning, decision-making, and executing them.

VI. CONCLUSION

LP is a branch of mathematics and statistics that permits researchers to find out the solutions to the problems of optimization. It is broadly applied to determine the optimal value in various real-life applications. It can be used in diverse fields of real-life situations such as transportation, production, management, technology, telecommunications, engineering, economics, industries, agriculture, and other issues. It helps to manage everything in a proper way whether it is physical or human resources. Similarly, it helps to maximize fairness of allocation, proper time management, scientific distribution of the product and it also secures the least-cost combination of inputs. It is a tool that can help for the best possible use of the limited resources to get the optimum production. It is also used to implement an optimal shipping plan for the distribution of a particular product from different manufacturing plants to various warehouses and solve several types of engineering design problems. Thus, it is mainly concerned with maximizing profit or minimizing costs through using appropriate production scheduling and inventory control, minimizing the wastage of raw materials, handing over the job to specialized personnel and applying proper transportation routes, etc. Thus, it helps to make the exact and quality decisions.

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CONFLICT OF INTEREST

Authors declare that they do not have any conflict of interest.

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